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seen in terms of taking the world further towards the brink of nuclear cataclysm; rather, the H-bomb would build a shield of such fearsomeness that war would become inconceivable. For many of those involved, the process of taking the United Kingdom into the H-bomb world was a profoundly constructive act.

The peace movement, emerging as a powerful political force although barely covered in this book, argued precisely the opposite. In 1950, 100 scientists petitioned the British government not to develop an Hbomb. India led a world campaign in favour of a ban on nuclear tests. The Pope condemned the H-bomb. By January 1956, the British prime minister, Anthony Eden, felt compelled to broadcast a message to the nation on the need to keep Britain's options open. And by the mid- and late 1950s, as Christopher Driver detailed in the still-classic The Disarmers: A Study in Protest (Hodder & Stoughton, 1964), these were matters of loud and passionate public and private debate; but one would not know that from Britain and the H-bomb.

Arnold is perhaps on stronger ground in examining the morality of the means than in justifying the official view of the moral imperative of the need for the H-bomb. Many technical questions are left to a fivepart appendix, for those who wish to learn more of the scientific argument. One of these sections focuses on evidence of the health and safety records of the tests. But little is learnt about concern for personnel weighing the balance of risk against the prospect of developing a technology 'to end wars' — or of the fate of the Christmas Islanders.

Britain and the H-Bomb tells the tale of the men who developed Britain's thermonuclear device through the drama of the tests in the Pacific. It does so with clarity and skill. However, some of the wider political and ethical questions still loom large.

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#### More on nuclear weapons

*Fallout: Hedley Marston and the British Bomb Tests in Australia* by Roger Cross (Wakefield Press, 2001; distributed in the United Kingdom by the Airlift book company) gives an Australian perspective on the British A-bomb tests conducted in Australia in the 1950s and their most vociferous opponent there, the fascinating but flawed biochemist Hedley Marston.

*Otto Hahn: Achievement and Responsibility* by Klaus Hoffmann, translated by J. Michael Cole (Springer, 2001), describes the work and thought of one of the discoverers of nuclear fission, and his reflections on scientific and social responsibility.



## **Fossilized art**

These pictures of fossils from the Triassic era, some 232 million years ago, were painted by Barbara Page and are from *Rock of Ages, Sands of Time* (University of Chicago Press, \$45, £28.50).

illustrated history of the Earth, with each contiguous panel representing one million years, going from the Cambrian to the present.

With text by Warren Allmon, the book is an

# Counting on the metaphorical

#### Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being

by George Lakoff & Rafael E. Núñez Basic Books: 2000. 451 pp. \$30, £21.99 (hbk), \$20, £14.99 (pbk)

#### **Gerald A. Goldin**

Modern cognitive science has blossomed in the past few decades. Might its ideas offer us a novel perspective on mathematical thinking, or even on what mathematics itself is?

How do mathematicians reason when they are defining new constructs or exploring abstract ideas? What makes certain ways of thinking rational or logical? If mathematics does not consist of universal truths, what accounts for its remarkable power, apparent timelessness and cross-cultural validity? Why does it describe nature so well? Does it consist of ideas and intuitions, or of theorems and formal proofs? And why do so many students struggle with mathematics, lacking real understanding of why they are manipulating those symbols and formulae?

George Lakoff and Rafael Núñez, linguist and philosopher respectively, answer these questions in a manner surely intended to provoke controversy. They use three main ideas, which they call cognitive science's "recent discoveries about the nature of mind". First, 'the embodiment of mind' is the idea that our bodies and brains, together with our experiences of the everyday world, structure our concepts and reasoning. Second, 'the cognitive unconscious' is the notion that essential aspects of our thinking, including low-level processes and systems of concept images and relationships, are not accessible to awareness. Finally, 'metaphorical thought' is the idea that we understand abstract concepts concretely in terms of our bodily experiences of sensation and movement, through a mechanism called "conceptual metaphor".

Actually, only the third idea is recent — Lakoff himself, together with Mark Johnson, has developed and vigorously propounded bodily-based conceptual metaphor as a near-universal explanatory construct, accounting not only for the language we use but also for how we think about space, time, life, love, good and bad feelings, and much else. Although it is far from being fully accepted in cognitive science or linguistics, the theory is presented in the book as if firmly established.

With these tools to hand, the authors take apart some of the most important ideas in mathematics. Non-mathematicians will find many of the explanations difficult, but should be able to grasp the general direction of the discussion. Topics covered include symbolic logic, sets and hypersets, transfinite numbers and infinitesimals, fractal curves, Dedekind's construction of the real numbers and Weierstrass's formal definitions in calculus. The book culminates in detailed "case studies" of e (the base of the natural logarithms), of i (the square root of -1), and of Euler's famous formula  $e^{pi} + 1 = 0$ . For each idea, the authors give us a conceptual metaphor that lies behind the mathematics; that is, imagery that grounds the concept in everyday experience (a "grounding

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metaphor") or that links it to another domain of mathematics (a "linking metaphor").

Several dozen conceptual metaphors are introduced, and each is given a proper name. Sometimes these are just new names for familiar notions, and add little. Thus, the 'Measuring Stick' metaphor lets us associate physical lengths with numbers, and the 'Numbers are Physical Segments' metaphor permits the opposite association. According to the "cognitive unconscious" notion, mathematicians typically do not realize that they are thinking metaphorically. For instance, in defining operations on functions with numerical values, they tacitly use the 'Functions are Numbers' metaphor. Complicated constructs such as the Cartesian plane (with x and y coordinate axes) involve "conceptual blends" of metaphors.

The most interesting and fully developed example of a conceptual metaphor is the 'Basic Metaphor of Infinity'. Lakoff and Núñez propose this as a general mechanism of cognition that originates outside mathematics, grounded in our everyday experiences with repeated processes (ordinary actions and movements) that come to completion. They call these experiences the source domain. They contend that mathematicians extend these experiences metaphorically to describe "iterative processes that go on and on" — the target domain. Whereas the source domain has a concrete, unique final state, in the target domain the final state is metaphorical and is called "actual infinity". The metaphor conceptually maintains the uniqueness and finality of actual infinity. By invoking the Basic Metaphor of Infinity, the authors describe as metaphorical a wide variety of mathematical concepts - from proof by induction and transfinite arithmetic, to the symbol • that is used to write formal infinite series.

As a survey of ideas in mathematics, this book does not compare favourably with other popular expositions. Occasional misconceptions, and frequent imprecision of mathematical language in otherwise valid explanations, make close page-by-page reading frustrating. Of course, the authors are not mathematical scientists. Like students new to the subject who are striving to understand the ideas behind formal mathematics, they "discover" that multiplication by *i* implements a 90° rotation, that space-filling curves do not fill space (when hyper-real coordinates are included) and that symbolic logic is "not absolutely true". Such interpretations, while not original, are the best parts of the book. But Lakoff and Núñez write as if they are the first to discover them, calling them "not new mathematical results, but new ways of understanding well-known results". They seem unaware that these and similar ideas are commonly used by good teachers of mathematics. And they often seem to assume, quite unjustifiably, that each mathematical construct can

be understood in only one such way — the one they have discovered — and that they have found the *real* metaphor from which the mathematics originates.

Lakoff and Núñez make many far-reaching claims based on their conceptualmetaphor analysis. They regard their work not merely as explaining imagery in mathematical thinking, but as profoundly affecting mathematics itself. They claim to have overturned what they call "The Romance of Mathematics" - conventional beliefs they attribute to most mathematicians, such as "mathematical truth is universal, absolute, and certain", and "the book of nature is written in mathematics" — and to have sketched, for the first time, what mathematics really is. They see their philosophy of "mind-based" or "embodied" mathematics as inconsistent with any existing philosophy, and "mathematical idea analysis" as a new discipline they themselves have launched, dethroning the queen of the sciences. Mathematicians sceptical of their ideas, they suggest, are likely to be Platonists (who believe in ideal forms), naive realists or empty formalists, influenced by self-interest, elitism and possibly a sense of wounded identity.

The arguments they offer as to why mathematics is not absolute are mostly familiar, although presented as novel. Their philosophical direction, which relies on our having access only to mathematics developed through human cognition, resembles well-known arguments for the essential subjectivity of all human knowledge. And the arguments are directed against naive, stereotypical opinions; serious philosophers are neither quoted extensively nor challenged. The profound evolution of mathematicians' understanding of 'truth' and 'existence', stretching across millennia, is omitted entirely.

I was most troubled by the narrowness of the book's base in cognitive science. No relation is acknowledged to most other work involving sensorimotor experiences, concrete and abstract understanding, or imagery in mathematics. There is some discussion of the brain, of arithmetic in animals and human infants, and of schemas and cognitive operations, but almost nothing about learning, human developmental and cognitive psychology, or problem-solving heuristics and strategies. Essential ideas of cognitive science, such as analogical reasoning, systems of internal representation, information-processing models, developmental stages, cognitive structures, or affect and motivation, find no place. Rather, "conceptual metaphor" subsumes or excludes all other constructs. By treating everything as metaphor - mathematical statements, definitions, proofs, representations and models, as well as notations, images, analogies, generalizations, mappings, conceptualizations and examples -Lakoff and Núñez disregard cognitively

important distinctions. And the notion of metaphor itself loses explanatory power.

In short, although I strongly favour analysing the ideas and imagery of mathematics, and agree with the authors' view that the "portrait of mathematics" has a human face, I regard this book as fundamentally flawed. Its flaws are independent of the straw opponents its authors set up. *Gerald A. Goldin is in the Departments of Mathematics and of Physics and Astronomy, Rutgers University, New Brunswick, New Jersey* 08903, USA.

# Sticking by our one and only?

#### The Myth of Monogamy: Fidelity and Infidelity in Animals and People

by David P. Barash & Judith Eve Lipton Freeman: 2001. 288 pp. \$24.95, £18.99

### T. R. Birkhead

In the mid-1970s, David Barash was one of the first biologists to embrace the new science of sociobiology and to look at adultery from an evolutionary perspective. He studied bluebirds, which, like most birds, were assumed to be monogamous. Barash wanted to know whether fear of female infidelity and the potential loss of paternity shaped



Birds do it, bees do it — even blue-footed boobies do it. But do they do it monogamously?