largely intractable, we are setting up a much more pressing problem for the future.

A probably still more important factor is that the book repeatedly speaks of a 2050 world in which huge numbers of people will still be in developing countries. Is this realistic? By 2025, China may well be the world's biggest economy, at least in terms of purchasing power parity. Already, at least 900 million people in so-called developing countries qualify as 'new consumers' with semi-affluent lifestyles. The perceived divisions of the world are becoming ever more blurred.

But despite these caveats, overall the book is an authoritative assessment of what we can expect for population totals at global, regional and national levels over the next half-century. It concludes that, for the most part, this will be a period when rapid population growth will slow to a virtual halt. The discussion of analytical methodologies is especially helpful to the non-demographer. The book might, however, have done more to stress that these are only demographic projections, and take no account of other factors, such as environmental constraints. Ethiopia has 65 million people today, many of them struggling to survive. Is it not likely that the country's mortality rate could rise, rather than decline as implied by most projections? Is it reasonable to expect that, within two generations, Ethiopia's population will grow to its projected 169 million? Perhaps it will, but the book should have highlighted potentially constraining factors.

The second book, *Population and Climate Change*, does indeed do just that. It considers environmental factors in terms of climate change, whether caused naturally or anthropogenically, and shows how climate change can induce demographic change, and vice versa. The fourteenth and fifteenth centuries featured unusually heavy rainfall and a high incidence of disease in Europe. Crop blights led to increased sickness in humans, with the result that the average longevity in England dropped within a century from 48 years to 38 years. The Black Death may well have originated in China or Central Asia during an exceptionally wet phase around 1332.

Today we are no longer so susceptible to most forms of established or conventional climate change, thanks to technological advances. But suppose that global warming causes the Gulf Stream to fail, as has been proposed in certain scenarios. This would profoundly disrupt agriculture, among other activities, in much of Western Europe. And the possible increase in the number and intensity of hurricanes along Florida's coasts would come on top of a fivefold increase in the state's population since 1950. Some 80% of today's populace lives within 35 kilometres of the coasts.

These and many other links between climatology and demographics are dealt with in an illuminating way by the three authors, who are all at the International Institute for Applied Systems Analysis in Vienna. A first-rate exercise in multi-disciplinary analysis, the book examines a host of associated issues such as fertility shifts, population ageing, agriculture, energy consumption fostering greenhouse-gas emissions, health patterns and environmental security, together with the many policy options they entail, especially institutional adaptations. It amounts to a timely exploration of the major challenges ahead. ■

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A scientist to count on

The Hilbert Challenge: A Perspective on Twentieth-Century Mathematics
by Jeremy J. Gray
Oxford University Press: 2000. 328 pp. £20, $34.95

W. Timothy Gowers

It will come as news to many that the year 2000 was World Mathematical Year. Why was this great event not more widely recognized? Partly, of course, because mathematics is a minority interest, but also because mathematicians themselves did not agree on a single official way to celebrate. Several conferences claimed to be the main occasion on which leading mathematicians would take stock of their subject and predict its future, and there is more than one book with a similar purpose.

A century ago, mathematical futurology was dominated, indeed almost invented, by one person. At the age of 38, the German mathematician David Hilbert gave a lecture at the 1900 International Congress of Mathematicians in Paris. In it he posed 23 problems, not all original to him, "from the discussion of which", as he put it, "an advancement of science may be expected". His lecture turned out to be extraordinarily influential — today's millennial conferences have all alluded to it, and could hardly have done otherwise.

One of the many virtues of Jeremy Gray's book about the Hilbert problems is that it convincingly explains how one relatively young man, giving one lecture — which, we are told, did not make much of an impression on its immediate audience — could have had the effect he did. One reason is that Hilbert's problems were very well chosen. He intended them to be difficult, but not so difficult as to be unapproachable, and, with one or two exceptions, his judgements were correct. Another reason is that Hilbert had worked in several areas — indeed, he is often held to be the last truly universal mathematician — so he was able to choose problems that spanned most of mathematics. A third reason is simply that the idea of presenting the world with a list of carefully selected unsolved problems was a brilliant one. Had Hilbert taken a more obvious path, such as outlining important programmes for research, he would not have captured the imagination of twentieth-century mathematicians in anything like the same way.

An important point that Gray brings out very well is that many of Hilbert's problems were linked by underlying philosophical and metamathematical themes. In other words, they were chosen not just for their intrinsic interest, but also for what they might reveal about the nature of mathematical argument itself. For example, it is a well-known fact of three-dimensional geometry that if two tetrahedra have the same heights and the same triangle as their base, then they have the same volume. But the only known proofs of this fact use calculus, or at least a limiting argument of some kind. By contrast, the equivalent two-dimensional theorem concerning areas of triangles can be proved by

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Galileo's Daughter
by Dava Sobel
Penguin, $14
"This is a harrowing tale of two victims: Galileo Galilei, forced to deny the evidence of his telescopes, and his daughter Virginia, forced at the age of 13 to enter the living sepulchre of the cloistered convent. Dava Sobel's latest well-timed bestseller provides a woman's angle on history's greatest conflict between science and religion." Brenda Maddox, *Nature* **403,** 702–703 (2000)

cutting one triangle into a few pieces and rearranging them to form the other one. Hilbert's third problem asked whether such a proof could be devised for tetrahedra. But his real interest was in the metamathematical question of whether the use of calculus was necessary. This was the first of his problems to be solved. In 1902, Max Dehn showed that calculus was needed.

Two of Hilbert's problems have, famously, had metamathematical solutions. His first problem was to prove or disprove Cantor's continuum hypothesis, which is the statement that there is no infinite set larger than the set of positive integers but smaller than the set of real numbers. Thanks to Kurt Gödel (in 1938) and Paul Cohen (in 1963), it is now known that this statement can be neither proved nor disproved. Hilbert's tenth problem asks for a systematic method for deciding which Diophantine equations have solutions. (Diophantine equations are polynomial equations whose solutions are required to be integers.) Building on the work of many mathematicians, Yuri Matiyasevich proved in 1970 that there was no such method. Results such as these have had a profound effect on the philosophy of mathematics.

The author of any mathematical book aimed at the general reader has to decide what background knowledge to assume, and Gray, like many others, is not consistent in his demands. This can be seen from a quick inspection of his 'boxes', those receptacles much loved of popular science publishers, which contain illustrations and (necessarily inadequate) explanations of some of the technical points in the text. That said, it would be misleading to describe this book as popular science. Two indications of its serious intent are that its title does not make silly use of the words 'history' or 'biography', and that we learn next to nothing about Hilbert's personal life. (For example, I still do not know whether he ever married.) As for the intended readership, at least some exposure to university-level mathematics is essential to appreciate the book properly.

My one complaint (apart from a few minor quibbles) is that Gray's prose contains far too many clumsily constructed sentences that I had to read twice. Here is one example from a long list: apparently, Hermann Minkowski thought it "unlikely that any polynomial in several variables which was never negative was expressible as a sum of squares". Surely, several of them are, one wonders, before realizing that the word "any" is supposed to be understood, unnaturally, as "every". This sort of writing lessens the pleasure of reading the book, which nevertheless remains illuminating and highly recommended. ■

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Going one better than nature?

Enriching the Earth: Fritz Haber, Carl Bosch, and the Transformation of World Food
by Vaclav Smil
MIT Press: 2001. 339 pp. $34.95, £23.95

John Emsley

The greatest catastrophe that the human race could face this century is not global warming but a global conversion to 'organic' farming — an estimated 2 billion people would perish. That is the underlying message of this remarkable book, which charts the discovery of nitrogen fixation — the conversion of unusable atmospheric nitrogen to useful ammonia — and its impact on the world's food supply.

If crops are rotated and the soil is fertilized with compost, animal manure and sewage, thereby returning as much fixed nitrogen as possible to the soil, it is just possible for a hectare of land to feed 10 people — provided they accept a mainly vegetarian diet. Although such farming is almost sustainable, it falls far short of the productivity of land that is fertilized with 'artificial' nitrogen; this can easily support 40 people, and on a varied diet. Of course, 'organic' farming should be encouraged in order to recycle compost and dung. But it can never compete with the bountiful supply of agrochemical nitrogen, which now meets about 40% of the world's dietary needs.

Nitrogen is abundant in the atmosphere, but in a form that is difficult to extract; only a few microbes and plants have the capacity to do this. Yet, thanks to their efforts over aeons of time, a whole planetary ecology can now be sustained. This organic nitrogen will even

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... And Carl Bosch: had faith that the process could be made to work commercially.

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Fritz Haber: discovered a way of converting nitrogen in the atmosphere into ammonia ...

support continued agriculture if properly managed, but it imposes a maximum on the density of the human population.

All this changed on 3 July 1909, when two German chemists, Fritz Haber and Carl Bosch, proved that it was possible to convert atmospheric nitrogen into ammonia on an industrial scale. Today there are Haber–Bosch chemical plants around the world, producing 150 million tonnes of ammonia a year, most of which goes into making fertilizer. The nitrogen input into farmed land from these fertilizers now exceeds the natural input. Even low-income countries can afford Haber–Bosch factories, and these should begin to turn around food production there, just as they did in high-income economies.

In the final chapter of *Enriching the Earth*, Vaclav Smil of the University of Manitoba admits that he originally intended to write a biography of Haber and Bosch, but he quickly realized that an account of the effects of their research would be far more interesting, and concentrated on this. He was right to do so.

Smil begins by looking at the fact that all living things need nitrogen in order to make amino acids, the building-blocks for the proteins on which life depends. He explains how nitrogen is fixed naturally, and how traditional farming takes this from the soil, but with only partial success at returning waste material to fertilize future crops. The first successful nitrogen fertilizers came from the Chilean guano deposits in the nineteenth century, a clearly limited supply.

The central theme of *Enriching the Earth* tells of Haber's struggle to make hydrogen gas (H_2) react directly with nitrogen gas (N_2) to form ammonia (NH_3), and of Bosch's faith that the process could be made to work commercially. Bosch then convinced the German chemical company BASF to invest in it. Thus was an industry born. But it was not immediately seen as the answer to the world's food