$1/d \approx 0.55$. Both these results do not necessarily contradict the results of Lin *et al.* but do illustrate the difficulty of unambiguously interpreting light-scattering data.

Lin *et al.* made all their new measurements at a wavelength of 488 nm, well away from the optical resonance and thus should lead to a reasonable estimate of fractal dimension. In earlier work, lower values of the DCLA fractal dimension were reported which may be attributable to neglect of optical resonance and multiple scattering effects. Dynamic light-scattering data can be ambiguous because aggregating systems contain broad size distributions of randomly shaped aggregates. For these systems the technique measures an apparent radius dependent on scattering vector, because rotational motion of larger aggregates distorts the result. Lin *et al.* use a scaling approach based on multi-angle dynamic light scattering to estimate a true radius whereas Wilcoxon *et al.* use a simpler low-angle approach which may lead to distortion (in fact their data agree with uncorrected data from Lin's group).

The paper by Wilcoxon *et al.* does, however, highlight the earlier inconsistencies of Lin *et al.*, a typical problem of rushing into print in a rapidly moving field. Although Lin's group may need a little chiding for having in the past published interpretations and results that had flaws, the present work convinces me of universality in colloid aggregation. But there are exceptions to every rule.... □

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MATHEMATICS

Mock theta conjectures

Ian Stewart

ONE of the most unusual people in the annals of mathematical research is Srinivasa Ramanujan, a self-taught Indian mathematician whose premature death left a rich legacy of unproved theorems. Ramanujan was preeminent in an unfashionable field — the manipulation of formulas. He tended to state his results without proofs — indeed on many occasions it is unclear whether he possessed proofs in the accepted sense — yet he had an uncanny knack of penetrating to the heart of the matter. Over the years, many of Ramanujan's claims have been established in full rigour, although seldom easily. The most recent example, the 'mock theta conjectures', is especially striking, because the results in question were stated in Ramanujan's final correspondence with his collaborator Godfrey H. Hardy. The conjectures have recently been proved by Dean Hickerson (*Invent. Math.* **94**, 639–660; 1988), following fundamental work of George Andrews and F.G. Garvan (*Adv. Maths* **73**, 242–255; 1989).

The word 'theta' refers to a remarkable range of special functions, known as theta functions, investigated at great length by Carl Gustav Jacob Jacobi (1804–51). They arose during the nineteenth century and rapidly became a major research area. Attempts to calculate the length of a general arc of an ellipse using integral calculus lead to simple-looking expressions which obstinately refuse to yield an explicit integral in terms of known functions such as polynomials or trigonometric functions. The reason turns out to be that these integrals require a genuinely new breed of function, called 'elliptic' functions. In a sense, elliptic functions are generalizations of the trigonometric functions, because an ellipse is a generalized circle and arc-lengths of circles involve trigonometry. But they have a deeper mathematical property. Trigonometric functions are periodic, that is, they repeat their values if a constant, the period, is added to the independent variable. The period of the sine function, for example, is 2π, because $\sin(x+2\pi) = \sin x$. Elliptic functions are doubly periodic: they have two independent periods, which in general are complex numbers, not real ones. This property singles them out and renders them worthy of serious study as pure mathematics, whereas their many applications to problems in number theory and dynamics make them useful enough to earn their keep in a competitive entrepreneurial environment. Although they have all but disappeared from the undergraduate mathematics course, they remain important at the frontiers of research.

Jacobi introduced four associated functions, the theta functions, which in the words of Morris Kline (*Mathematical Thought from Ancient to Modern Times*; Oxford University Press, 1972) are "the simplest elements out of which the elliptic functions can be constructed". He also obtained expressions for them in the form of infinite series and infinite products and proved a number of remarkable identities.

Ramanujan's last letter to Hardy (S. Ramanujan in *Collected Papers*, 354–355; Cambridge University Press, 1927) introduces ten new functions, occurring in two groups of five. They are defined by series which in some respects are similar to Jacobi's, and Ramanujan asserted that they share many analogous properties, though, as usual, he gave no proofs. They are accordingly known as mock theta functions and said to be of the 'fifth order' to distinguish them from related functions found elsewhere in Ramanujan's works.

But are they genuine mock theta functions, rather than ordinary Jacobi theta functions in some as yet unpenetrated disguise? Real fakes rather than fake fakes? It is a subtle question, but an important one. If Ramanujan's mock thetas are really real thetas, they immediately lose any interest. Of course fancy combinations of genuine thetas behave like genuine thetas. As with certain art forgeries where the original artist has gone out of fashion but the forger has acquired notoriety, only a genuine fake is interesting. To put it another way: real thetas are interesting, and mock thetas share their remarkable properties, so provided that they are really new, mock thetas are also interesting.

And that is where the mock theta conjectures come in. Ramanujan recorded his results in a series of notebooks. One temporarily went missing, becoming the romantic 'lost notebook', but eventually turned up safe and sound. Following early work of G.N. Watson, George Andrews discovered a formula in the lost notebook which, if true, means that the mock thetas are genuine fakes. In his paper with Garvan cited above, he shows that this formula is equivalent to two slightly involved statements in number theory. These concern the partitions of a given integer — the ways of writing it as a sum of smaller integers. The original papers should be consulted for the details.

Hickerson has now proved these number-theoretic conjectures, so the mock thetas are genuine fakes and are therefore interesting new functions worthy of further study. The proof involves delicate manipulations of infinite series of a kind that would have delighted Ramanujan. The astonishing complexity of the proof underlines, yet again, the depth of Ramanujan's genius. It is very hard to see how anyone could have been led to such results without getting bogged down in the fine detail. Hickerson has extended his methods (*Intent. Math.* **94**, 661–677; 1988) to tackle the seventh order mock theta functions, also introduced by Ramanujan and equally enigmatic. They too prove to be genuinely mock.

Ramanujan was the formula man *par excellence*, operating in a period when formulas were out of fashion. Today's renewed emphasis on combinatorics, inspired in part by the digital nature of computers, has provoked a renewed interest in formal manipulations. The half-forgotten ideas of Srinivasa Ramanujan are breathing new life into number theory and combinatorics. □

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