Making the geoid respectable again

With the recognition that surface measurements of gravity cannot be extrapolated backwards into the Earth's interior, the concept of the geoid became unfashionable. But its fortunes may be on the turn.

EVERYBODY knows, of course, that the geoid is that surface of equal gravitational potential which coincides with mean sea-level. Most of us also have it clearly in mind that the geoid is an ellipsoid with rotational symmetry about its shorter polar axis, which allows for the flattening of the Earth by rotation. But that must plainly be a fairy story, possibly applicable to the oceanic surfaces of the Earth, but unlikely to make very much sense elsewhere. The snag, as can be learned from a glance at a recent article by R.G. Hipkin from the University of Edinburgh (*Geophysical Journal* **92**, 53; 1988), is that it is much easier to stick to the fairy-story than to tell just where the geoid is on dry land.

Hipkin's opening sentence may be a sufficient explanation:

> Physical geodesy has become a dauntingly esoteric branch of applied mathematics which, in the achievement of greater rigour, has distanced itself from many practical geodesists. This is particularly unfortunate when greatly improved measurement techniques have become available, both for surface gravimetry and for geometrical positioning from space vehicles.

Hipkin might have added that geodesy may not be the only offender in this regard.

Whatever the validity of that *canard*, there is clearly plenty of room for intricate argument about even the elementary definitions on which geodesy is founded. Hipkin's approach is refreshingly that of a logical positivist of the machian school. In pursuit of the principle that the only quantities appearing in equations should be measurable quantities, he even rejects Newton's scheme (really, a *gedanken-experiment*) for defining the geoid everywhere by building a series of wells and canals within the body of the Earth; the construction would vitiate the observations, he says.

With that said, Hipkin's interest is to rescue the concept of the geoid from the limbo into which it may be expelled by excessive rigour. (With tongue firmly in cheek, he quotes two writers from twenty years ago who say that this ambition is a "concession to conventional conceptions"). But the difficulties are considerable, as Hipkin's article shows clearly enough.

Were it not for the continents, and anomalies of gravitational potential such as that in the south-west Atlantic or the non-circularity of the Equator, the surface of mean sea level would indeed be an ellipsoid of rotation. One of the practical difficulties with which geodesists are confronted is that the measurements of sea level at tidal stations must be corrected for such sources of variation as ocean currents, departures of seawater density from the means, not to mention other seasonal effects, but in principle there is a wealth of places scattered around the world at which the ellipsoidal parts of the geoid might be defined precisely, especially now that satellite ranging data may provide locations more precisely (or at least more conveniently) than by timing stars.

One superficial complication is that the reference ellipsoid and the set of concentric ellipsoids corresponding to different values of the gravitational potential define a downward gravitational attraction (called "normal gravity" and calculated as $\gamma = \nabla U$, where ∇ is the laplacian operator and U is the potential field) that will differ both in magnitude and in direction from true gravity, $\mathbf{g}$ (notionally calculable from the true gravitational potential W as $0.179W$).

Hipkin does not differ from others in remarking that it is in principle possible to relate measurements of gravity (including the departure of its direction from that of the line to the centre of the Earth) at inland surface sites to measurements at coastal tidal stations by a combination of levelling and gravity measurements (but care is needed to distinguish between the normals to the ellipsoidal surfaces and the direction of a plumb line).

The essence of the difficulty is that of telling what happens to the force of gravity beneath the surface, within the continental crust for example. Formally, nothing can be done without a knowledge of the density distribution of material within the body of the Earth. While, as every schoolboy knows (or, perhaps, should know), the external gravitational field of the Earth is uniquely determined by a knowledge of its value and gradient at each point on the surface, nothing can be said about the gravitational field within the Earth without a detailed knowledge of the density distribution within the solid Earth. That seems to be the starting point, apparently established by the Soviet mathematician Molodensky, for the recently rigorous trend in geodesy.

Plainly, there are also serious mathematical problems in extrapolating downwards from the surface of the Earth even when there are ample measurements of gravity on which to base inferences. Hipkin gives the example of a ploughed field, the furrows of which would yield small but still measurable periodic variations of gravitational potential just above the surface which, if projected downwards in a simple-minded fashion, would imply variations of the height of the equipotential surface of as much as 1 km just a few metres below the surface.

Hipkin's goal is to find a sensible and consistent way of projecting surface measurements of gravity downwards. This he does by splitting the measured gravity anomalies at the surface into two parts — that due to variations from flatness of the terrain in the immediate neighbourhood and that represented by what is usually called the Bougier anomaly, and which corresponds to deep-seated gravity anomalies such as the departures from hydrostatic balance, more precisely isostacy.

The proof of the recipe is, as always, whether it works. On the face of things, and for a careful set of data collected in northern Scotland, the answer is encouraging. First, it appears practicable to deal with the effects of topographic variations (of which there are many, called mountains, in northern Scotland) by making the simple assumption that they consist of material of constant density and then by estimating their contribution to local gravity by means of a contour-following technique. Second, the downward projection of the Bougier anomaly itself can be arranged in a consistent fashion by careful control of the wavelength of the measured surface gravity field.

The result, for northern Scotland, is a more detailed account of where the geoid lies than any other part of the world is provided with. On the face of things, the height of the geoid there is defined with a precision of less than 10 cm. These variations become apparent after allowing for an east–west tilt in the surface of the geoid from one side of Scotland to the other. In principle, and in due course, this should provide grist for the mills of those wishing to speculate about the reasons why, for example, there should be a relative depression of the geoid amounting to a maximum of more than 1.5 m off the northern coast of Aberdeenshire. But the general interest of what Hipkin has done is to make the concept of the geoid respectable again.

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