New ways with quasi-crystals

While theorists continue to brood about the properties of quasi-crystals, experimentalists have begun to simulate them in the laboratory to tell how they behave.

WHETHER the unexpected fivefold geometrical symmetry of the manganese-aluminium alloy $MnAl_{13}$ springs from an underlying quasi-crystalline structure, from twinning on a microscopic scale as Linus Pauling says (see *Nature* **317**, 512; 1985) or from a quite different phenomenon not yet identified, the whole idea of fivefold symmetry has enlivened several groups of people. That the theorists should have jumped in the new pool feet first is not at all surprising, which is why it is a little shocking that there is still no straightforward way of writing down a formula that will specify which points in three-dimensional space are the permitted vertices of a quasi-crystalline lattice. (The best approach is still to suppose that a three-dimensional quasi-crystal is the projection of a six-dimensional lattice on a three-dimensional surface.) But now the experimentalists have joined the fray, with entertaining consequences.

That fivefold symmetry is unattainable in an infinite regular crystal lattice has been known since the allowable symmetries of crystal lattices were first enumerated in the nineteenth century. The quasi-crystal explanation of the $MnAl_{13}$ alloy, whose strange symmetry was first recognized by Schachtman, remains essentially that suggested more than eighteen months ago by D. Levine and P.J. Steinhardt (*Phys. Rev. Lett.* **53**, 2477; 1984). A quasi-crystal is not a regular lattice in which all vertices can be reached from all others by displacements represented by integral multiples of some basic set of vectors. Instead, the argument goes, the directions of the lattice bonds drawn between nearest neighbours in the crystal lattice take only a finite number of values, but the distances between successive vertices along a line, far from being equal, as in a regular crystal, may take one of two (or more) values which are, of necessity, incommensurate — their ratio is an irrational number. In two dimensions, the prototype of a quasi-crystal is the Penrose tiling of the plane, a kind of party trick by which a plane surface can be filled by rhombi of two distinct shapes yielding local fivefold symmetry. In this case, the crucial irrational number is the old Greek golden mean, half the square root of 5 plus 1.

How do experimentalists weigh into such a field? Here is a neat example, based on the familiar observation in superconductivity that the magnetic flux within a sufficiently small superconducting loop will ideally be quantized in the sense of being an integral multiple of the elementary quantity $hc/2e$, where h is Planck's constant, c the velocity of light and e the charge of the electron. Only integral multiples of this magnetic flux are compatible with the condition that the superconducting loop carries no electric current. The origin of the condition is the requirement that the phase of the wave function describing electrons in the superconductor should be unambiguously defined.

Now a six-person group based mainly at the University of Pennsylvania but including Levine (*op. cit.*) has devised a clever experiment to show that magnetic fields can tell the difference between lattices that are regular and those that are merely quasi-crystalline (A. Behrooz, *et al.*, *Phys. Rev. Lett.* **57**, 368; 1986). What this group has done is to build a pair of two-dimensional arrays of aluminium wires merely 50 nm in diameter, one of which is a quasi-crystal of holes in one dimension, the other almost a Penrose tiling. The group has then measured the superconducting properties of this array in the presence of magnetic fields of different strength with the objective of telling how quanta of flux are distributed among the differently shaped holes of the lattice.

Others have previously set out to tell what happens when regular (not quasi-periodic) superconducting meshes are immersed in a magnetic field. If the total amount of flux, supposed uniform, across the mesh is an integral number of flux quanta, no electric current need flow in any of the wires so as to ensure that the phases of the electron wave functions are well defined. One of the simplest ways of recognizing that state of grace is to measure the temperature at which the superconductive transition takes place. If the temperature is identical with that of the superconducting phase transition in the absence of a magnetic field, the inference is that the flux through each and every hole in the mesh is an integral multiple of the flux quantum. Otherwise the transition temperature will be reduced. Behrooz *et al.* refer to earlier results showing that when the total flux through a regular mesh is less than enough to provide one flux quantum through every hole, the transition temperature is most nearly normal when the field is enough to endow each hole in the mesh with a rational (as distinct from irrational) fraction of a single flux quantum.

That is an obvious starting point for the experiments now described. The simplest trick is to make a mesh from 50 nm aluminium wires which are regularly spaced in one direction but whose spacing in the orthogonal direction is that of the Fabonacci sequence, whose spacings are related by the irrational golden mean. For practical purposes, there are holes of two different sizes whose areas are irrationally related to each other. The experimental result is striking, even startling. The temperature of the superconducting transition is most nearly normal when the average magnetic flux through each hole amounts to a number of flux quanta which is a power of the golden mean. The inference is that the most energetically favourable distribution of the magnetic flux is one on which as many holes as possible are threaded by integral numbers of flux units, which is what would be expected. It is nevertheless remarkable that it should in principle be possible to measure the irrational ratio of the dimensions of the mesh simply by measuring the transition temperature as a function of total flux through all the holes.

Similar results have been obtained with a network analogous to the Penrose tiling of the plane, but where the characteristic irrational ratio is the square root of 2. (Forgivably, the authors say that they were unable to make a true Penrose tiling with their microfabrication apparatus, which is nothing to be ashamed of; they managed 20,000 holes of two different shapes related by the square root of 2.) Again the most favourable threading of the total flux through the different holes is that determined by the irrational number. The principle is merely that the best configuration is that when the quantum condition is most often satisfied. What the practical applications will be is anybody's guess, but there could hardly be a more vivid proof of the quantization of magnetic flux.

As it happens, a calculation of a more general system of this kind, a randomly organized superconducting network, has been produced by two Argentinian physicists, J. Simonen and A. Lopez (*Phys. Rev. Lett.* **56**, 2649; 1986). Again, the conclusion is that integral numbers of flux quanta are energetically favoured. Meanwhile, experimentalists are busily studying other quasi-periodic structures. Obviously there is a whole new world to explore. **John Maddox**