xion' array (Buckminster Fuller) and all touch an inner sphere of the same radius.

The point is not trivial, for I have been constructing models of quarks made up of infinite dimensional spheres of unit radius (each has a content then of 47.123789). Twelve of these in icosahedral array surround a central infinite sphere with 0.734 of the three-dimensional content of the outer spheres. This results in a content, taken over infinite dimensions, of 600.08 (600 + (1/12)). This allows the central space to be a four-dimensional 600-vertexed regular polytope. A quark is proposed as having an infinite kernel of 47.123789 giving 647.123789 for the u and d quarks. In a proton only a three-dimensional icosahedron is present in the kernel giving a content of 3 × (600.08 + 12) = 1,836.24.

In the neutron each icosahedral array changes to a dymaxion array (making it unstable) surrounding another sphere giving 3 × (600.8 + 13) = 1,839.24

Some corroboration of this approach can be found in the examination of the content of different dimensional spheres, which rise to a maximum at five dimensions from unity at zero dimensions and reach unity again at 12.76 'fractal' dimensions[1].

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The distribution of charges in classical electrostatics

SIR—Berezin[1] has drawn attention to the "surprising" fact that the minimum-energy configuration of a system of N equal electric point charges q, constrained to lie on a disk of radius R, is in general not the arrangement with all the charges on the periphery with equal spacings (configuration A).

Berezin goes on to say that his result may have a bearing on truly three-dimensional geometries, but it can readily be proved that for a system of point charges confined to a spherical region, the minimum-energy configuration cannot conceivably have a charge at the centre.

Such a configuration would contradict Earnshaw's theorem[2] that a charge (say, at the centre of a sphere) cannot be held in stable equilibrium by the electrostatic field of all the other charges. This does not preclude Berezin's configuration B in the case of a disk, since a charge at its centre may be in stable equilibrium with respect to variations only in the plane of the disk.

Berezin's statement[1], that for N charges on the disk, one charge will be displaced to its centre (configuration B) for $N \geq 12$ is not valid.

With increasing N, the charges appear to arrange themselves in more and more complex patterns of concentric rings, with equal spacings on each ring. This problem was investigated by calculating numerically the total energy for various numbers of equally spaced charges on each of several such rings as a function of the radii of the rings, with a possible charge at the centre. The results are insensitive to the precise angular coordinates of charges.

With units in which $q = R = 1$ configuration B is that of minimum energy only for $12 \leq N \leq 16$. For $N = 17$, there is a lower-energy configuration with two charges at radius $r = 0.31$, the other 15 charges being at $r = 1$; its energy is $W = 133.8$, which is lower than the value $W = 134.1$ for configuration B.

When $N = 29$, for example, the lowest-energy configuration has six charges at $r = 0.49$, the remaining 23 being at $r = 1$ ($W = 444.5$, compared with $W = 459.4$ for configuration B). But when $N = 30$, the minimum-energy configuration has one charge at the centre, six at $r = 0.55$ and 23 at $r = 1$ ($W = 479.1$, compared with $W = 496.5$ for configuration B).

This trend suggests, as one might expect, that the charges tend to spread over the disk, though most of them always remain at the periphery. In fact, the continuum limit of an electrified conducting disk is known[3]. If the total charge is Q, the charge density at radius r is $\sigma(r) = Q\,(2\pi R(R^2 - r^2)^{1/2})^{-1}$.

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SIR—Berezin[1] considers how N charges would distribute themselves over the surface of a disk and seems surprised to find that, for $N \geq 12$, they do not all lie around the circumference. But in the limit of very large N, they would surely be distributed with surface density $\propto (a^2 - r^2)^{-1/2}$, where a is the disk radius (for $r < a$), this being the charge distribution over a conducting disk given by simple electrostatics. The charges would actually group themselves into an N-dependent lattice pattern: it would be interesting (and perhaps relevent to snowflake growth, for instance) to calculate what these configurations are.

Classical electrostatics does *not* (as Berezin implies) tell us that the charge lies only at the edge of a two-dimensional conductor. (The *charge density* does, however, tend to infinity at the edge.) In three dimensions, the charge does lie on the surface and a Berezin-type calculation would confirm this rather than requiring "modification of usual theorems of electrostatic stability". This is readily seen for the case of a conducting sphere or ellipsoid. Suppose that the surface were charged and that one extra test charge were at the centre. If the surface charges were held fixed, the test charge would feel no force even if it were displaced from the centre; however, the surface charges distribution would adjust (being repelled by the test charge), and the adjusted distribution would attract the test charge towards the surface, implying that no charge could remain stably in the interior.

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SIR—My thanks to those who wrote comments on my recent letter 'An unexpected result in classical electrostatics'[1]. The paradox is, indeed, resolved, if the circle with point charges is treated as a thin disk embedded in three-dimensional space and the central charge is actually at the centre of one of the flat surfaces of the disk. There will be, of course, no critical N for the 'charge ejection' to the centre of the case of a three-dimensional *empty* sphere. The problem of the gradual build-up of concentric spherical layers of point charges will, however, remain for the sphere containing a uniformly spread fixed positive charge (a static version of J. J. Thomson's atomic model).

This discussion has also revived an interest in the old but still largely unresolved problem of how to find the equilibrium configuration of N equal point charges placed on the surface of a sphere. Despite a number of valuable contributions[2-5], its solution is known only for some special values of N and the general algorithm valid for any integer N is yet to be found. Even for the 'simple' cases of N=4, 6, 8, 12 and 20 (platonic polyhedrons) there is a very interesting curiosity which seems to contradict commonsense. While the tetrahedron (N=4), octahedron (N=6) and icosahedron (N=12) do indeed provide the minimum energy configuration, the cube (N=8) and dodecahedron (N=20) do not[2,3,5]. It, perhaps, may be attributed to the fact that tetrahedron, octahedron and icosahedron all have 'rigid' triangle faces, while the cube and dodecahedron have 'soft' deformable faces (square and pentagon respectively).

Numerous other possibilities also remain open for study; for example, N non-equal charges (Q, $2Q$, $3Q$ . . .), an unequal number of positive and negative charges confined to two close concentric spheres, point charges on non-spherical surfaces, and so on.

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