must be a fairly good approximation to the distribution of a number of identical charges (say electrons) placed on the disk. It is not confined to the circumference and it is *not* zero at the centre. Hence there is really nothing unexpected in the results given by Berezin[1] who shows that 12 identical charges distributed evenly on a circle have a greater potential energy than 11 charges on the circle and one at the centre. Perhaps the fact that 12 is the precise number at which one charge is "expelled to the centre" is mildly surprising.

Berezin goes on to conjecture that a similar "spontaneous ejection" of charge to the centre will occur in three dimensions and that this "may lead to some modification of usual theorems of electrostatic stability which claim that at the state of equilibrium all charges . . . are always located at the surface." I think that this conjecture is wrong, and is a nice example of incorrect generalization from two to three dimensions. For example, arguing from two[2] to three dimensions about the propagation of an impulsive wave emitted from the origin would lead one to the false conclusion that there is a residual disturbance left at the centre in the three-dimensional case.

However, Berezin's letter raises a question to which I have been seeking an answer for some time. It seems clear that it is possible to place an arbitrary number, N, of electrons on a conducting sphere. The question is how do they arrange themselves?

There is little doubt in my mind that they will arrange themselves on the surface, and intuitively the answer would seem to be given by simple symmetry arguments when $N=1, 2, 3, 5$ and $4, 6, 8, 10, 20$, where the second set of numbers are the numbers of vertices of the five regular polyhedra in three dimensions. However, as Berezin has pointed out (in personal communication), such arguments are false in the case where $N=8$. Having the electrons at the vertices of a regular cube has a *higher* energy than when one face is rotated relative to the opposite face by 45°, the distance between the faces being held constant. A quantum treatment of this problem would also be interesting and could also yield mean values of the multipole moments of the system.

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2. Morse, P. M. & Feshbach, H. *Methods of Theoretical Physics*, 1363 (McGraw-Hill, New York, 1953).

. . . with charged polygons

Sir—The classical electrostatic energy of N point charges, q, symmetrically disposed on a circular circumference with radius R has been recently discussed here (A. A. Berezin, *Nature* **315**, 104; 1985). A surprising feature of these regular polygonal systems was reported for $12 \leqslant N \leqslant 400$. Such arrangements are energetically unstable with respect to displacement of one charge to the centre and regular circumferential redistribution of the remaining $(N-1)$ charges. On this basis Berezin made two conjectures: (1) that such a rearrangement is energetically favourable for any large N and (2) that his large-N result indicates a violation of the familiar electrostatic theorem. For a conducting body at electrostatic equilibrium, any net charge must be located at the surface (Faraday shielding).

It should be pointed out that conjecture (2) cannot follow from the large-N polygon results.

If these systems are to model a classical conducting system, their electrostatic potential energy should vary as: $W=\frac{1}{2}cq^2N(N-1)$ with c a geometrically determined constant (capacitance coefficient). This in turn requires that the electrostatic potential at any vertex location due to the *other* $(N-1)$ charges should be $U=cq(N-1)$. Evaluation of exact electrostatic potentials for polygons with $N=10^3$, 10^4, 10^5 yields respectively $U=2{,}238.7970$; $29{,}717.3261$ and $370{,}466.8178$ (units of q/R).

Clearly the required linear N dependence does not occur. This failure relates to the usual logarithmic distance dependence of electrostatic potentials near linear charge distributions. Thus the polygonal charge arrangements cannot model a conducting system.

For the problem at hand, the potential U may be approximated by exactly summing the contributions of the $2j$ charges closest to a vertex location and treating the remaining charges continuously. For example, choosing $j=18$ and N large:

$$U \simeq \frac{Nq}{R}\left[1.112527455 + \frac{1}{\pi}\ln\cot\frac{37\pi}{4N}\right]$$

This expression reproduces the potential energy, U, for polygons with $N \geqslant 10^3$ with fractional accuracy better than 2.3×10^{-5}; this accuracy improves with increasing N. The non-linear N dependence is manifest. Using such an algebraic approximation Berezin's first conjecture, the instability of any large-N polygon system, is readily confirmed.

For N surface charges regularly distributed on the surface of a sphere with radius R, the limiting form of the electrostatic energy is:

$$W = \frac{1}{2}\frac{q^2}{R}(N)(N-1)$$

Accordingly for the spherical surface, expulsion of one charge to the centre is energetically possible only in the infinite N limit.

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. . . but for 12 equal point charges?

Sir—Berezin has recently shown that the minimum energy configuration for N equal point charges placed in a circle is different depending on whether N is less than, greater than or equal to 12 (*Nature* **315**, 104; 1985). In the former case $(N<12)$ the configuration of minimum energy is with the N charges at the vertices of a regular polygon inscribed in the circle (configuration A); in the latter case $(N\geqslant 12)$ a lower energy configuration is obtained with one charge expelled to the centre of the circle and $(N-1)$ charges at the vertices of an $(N-1)$ sided inscribed polygon (configuration B). Berezin verified this result up to $N=400$ and claimed it was "very likely" true for all $N\geqslant 12$. It is easy to show the result does indeed hold for *all* $N\geqslant 12$ as follows.

The energy of N charges arranged in configuration A is

$$W_A(N) = \frac{N}{2}\left\{\sum_{i=1}^{\frac{(N-1)}{2}}\frac{1}{\sin \pi i/N} + 0.5\,(N \text{ even})\right\} \quad (1)$$

and arranged in configuration B is

$$W_B(N) = \frac{(N-1)}{2}\left\{\sum_{i=1}^{\frac{(N-2)}{2}}\frac{1}{\sin \pi i/(N-1)} + 2 + 0.5\,(N \text{ odd})\right\} \quad (2)$$

It follows simply from equations (1) and (2) that

$$W_B(N) = W_A(N-1) + (N-1) \quad (3)$$

Hence the condition that configuration B gives higher energy than configuration A $(W_B(N) > W_A(N))$ is true for those N where

$$W_A(N) - W_A(N-1) < (N-1) \quad (4)$$

By inspecting the numerical data for $W_A(N)$ only, looking say at the range $2<N<40$, it is possible to see that the inequality (4) is satisfied for $N<12$ as shown by Berezin. It is also easy to see that the left hand side of inequality (4) is increasing faster than $(N-1)$ as N runs beyond 12. Hence the result found by Berezin is true for *all* $N\geqslant 12$.

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