WHENEVER we assume that words probably mean roughly what they have meant in the past, and in particular whenever we write letters to *Nature*, we are relying on probabilistic induction[1,2]. Therefore, when Popper and Miller[3] claim to prove the impossibility of inductive probability this argument needs to be watertight if it is to be believed. Their argument falls short of achieving the potentially watertight (but unachievable) form stated in my next paragraph.

Let h be any hypothesis and e any event such that $p(e) \neq 1$ and $p(h, e) \neq 1$, and suppose that h can always be written in the form $h = ab$ where (1) e deductively implies b, (2) e probabilistically undermines a, that is $p(a, e) < p(a)$, and (3) a and b are probabilistically independent. All this is to be true even if $p(h, e) > p(h)$. This situation would I think be "completely devastating to the inductive interpretation of the calculus of probability". to quote Popper and Miller[3].

Popper and Miller in effect define a as $h \leftarrow e$ and b as $h \vee e$. Then all conditions are satisfied, except that their condition (3) is replaced by the weaker condition: (3a) a and b are probabilistically independent when e is given (they are also independent given $-e$). But, apart from the uninteresting cases in which $p(h, e) = 1$ or $p(h, -e) = 1$ or $p(e) = 1$ or $p(-e) = 1$ (where $-$ signifies not), the independence of a and b is impossible, that is, $p(a)p(b) \neq p(ab)$ which is $p(h)$. This can be proved by elementary algebra (the proof is available on request). Because I believe that inductive probability cannot be refuted I predict that Popper and Miller will not be able to redefine a and b to satisfy the conditions of my second paragraph, which include condition (3) instead of only (3a). Thus, in accordance with Popper's standard and justifiable requirements, I have stuck my neck out and have handed him an axe.

After reading the text above, David Miller asked me in what way I "understand the splitting of a hypothesis into an established and an ampliative part"; that is to say, "how are we to specify which part of a hypothesis goes beyond the available evidence e?" My definition would be the set H of those deductions from h that are not deductions from e. This definition is not the same as $-e \vee h$, or $h \leftarrow e$, which was the definition proposed by Popper and Miller. I think my definition has more intuitive appeal, and if it is used, Popper and Miller's argument against the possibility of induction would not apply.

Miller has asked further why I think that the part of h that goes beyond e should be statistically independent of e. The answer is that I do not by any means think this: I raised the issue of the probabilistic independence of $h \vee e$ and $h \leftarrow e$ because Popper and Miller stated in their original letter[3] that "given e, the factors of h are probabilistically independent" so I assumed that this was intended to be a link in their chain of reasoning. If it was not so intended, then the second and third paragraphs of the present letter can be ignored. Moreover, if my definition of the ampliative part, H, of h is accepted, then this issue of independence would again evaporate.

Miller has pointed out in correspondence that H is not a proposition, for it contains h, but does not contain $h \vee e$ even though $h \vee e$ is a deduction from h. He has pointed out further that H contains some statements, such as $h \leftarrow e$, that are undermined by e. These are interesting observations, but I still think that H, rather than $h \leftarrow e$, provides the natural meaning for the part of h that goes beyond e. Probabilistic induction might be complicated but it cannot be impossible for the reason given in the first sentence of this letter.

Note added in proof: If inductive support does not mean probabilistic support, I do not know what it means.

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1. Black, M. in *The Encyclopedia of Philosophy* Vol. 4, 168–181 (Macmillan, New York and The Free Press, Glencoe, 1967).
2. Good, I. J. *J. Statist. Comput. Simul.* **13**, 154 (1981).
3. Popper, K. & Miller, D. *Nature* **302**, 687–688 (1983).

POPPER AND MILLER REPLY—As Levi says, "probabilistic support speaks with many tongues"; in other words, between he, the strongest, and $h \leftarrow e$, the weakest statement to entail just h in the presence of e, there are many others, and each may have a different degree of probabilistic support given e. Let x be one of these statements, so that

$$he \vdash x \vdash h \leftarrow e$$

(where $\vdash$ signifies entailment). Then it follows simply from Jeffrey's equation (4) that

$$s(x, e) = s(x \vee e, e) + s(h \leftarrow e, e)$$

The first of the terms on the right is greater than or equal to zero, but purely deductive. So only the second term, which is never positive, can be said to be non-deductive (and so perhaps inductive). Inductive support (if any) is therefore at most zero: all non-deductive support is countersupport. None of our three correspondents has said a word against this thesis: Levi seems to agree; Jeffrey's f is a special case of our x; Good's letter begins and ends with a declaration of faith in probabilistic induction. In the rest of his letter (as he says, his second and third paragraphs can be disregarded) he proposes (like Jeffrey) a different factorization from ours. But the proof of our thesis does not depend on our factorization, only on the fact that $s(h \vee e, e)$ or $s(x \vee e, e)$ is purely deductive support; whilst $s(x \leftarrow e, e) = s(h \leftarrow e, e)$ is the only support without a deductive part that is obviously redundant in the presence of e.

Thus all probabilistic support that is not countersupport is purely deductive.

We wish to withdraw as incorrect the suggestion near the end of our letter of 21 April 1983 that $s(h \leftarrow e, e)$ invariably decreases as the content of e increases. Our main thesis is not affected by this correction.

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