Saturn's rings

MUCH of the interest in Saturn's rings during the impending close encounter of Voyager II will centre on explanations of the structure of the ring system, involving either resonance with the major planetary satellites or the effects of smaller satellites co-moving with the rings. The excitement may obscure the argument due to Joseph E. Avron and Barry Simon (*Phys. Rev. Lett.* **46,** 1166; 1981) published earlier this year that the radial structure of the rings of Saturn may simply reflect the instability at certain radii of solutions of the full many-body problem.

The argument is simple, although incomplete. Ignoring all but Saturn and one ring particle, supposed of negligible mass, orbits with all possible periods of revolution between that of the orbit that grazes the equator of the planet and infinity are stable. No more complicated version of the problem is, however exactly soluble. The general equation of motion is, however, $\ddot{\mathbf{r}} = \mathbf{F}(\mathbf{r}(t))$ where the force-function **F**, a vector function, itself entails a solution of the many-body problem of Saturn, together with its substantial satellites and, for that matter, the Sun.

Approximations are evidently necessary, one of which is that the true force-function should be represented as the sum of forces within the equatorial plane and those perpendicular to it — whereupon it is also sensible to represent the radius vector as the sum of a vector in the plane and another perpendicular to it with the form $w(t)$. To a first approximation, supposing w to be small, the result is the need to solve a Schrodinger equation with (at best) a quasi-periodic potential. This simplest version of the problem of the motion of Saturn's ring particles can, in other words, be approximated to by the still insoluble problem of knowing what states are accessible to an electron in a quasi-periodic electric potential.

The upshot is that many periodic solutions of the simplest equations are unstable and therefore, over a substantial period of time, untenable. But which are they?

Avron and Simon did not pretend, in their paper published at the end of April, to be able to calculate the periods of the orbits that would be unstable (and thus the position of the gaps in Saturn's rings). Instead, they argued that even the simplest approximation to the many-body problem gives rise to Schrodinger-like equations whose solutions are restricted. Some orbits are possible and stable. Others are not. Avron and Simon surmise that between the rings, stable orbits should be distributed in radius as the numbers in a Cantor set — sparsely but not isolated.

Solar spin variation

from David W. Hughes

ONE of the most striking features of normal main-sequence stars is the great difference in the angular momentum per unit mass between the early and late types. There is a sharp drop at spectral type F, as shown in Fig. 1. Some braking process must act preferentially on stars of low mass.

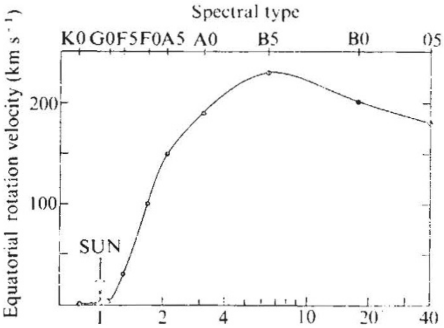


Fig. 1 The equatorial rotational velocity of main-sequence stars plotted as a function of spectral type and stellar mass (measured in units of the solar mass).

Schatzman (*Ann. Astrophys.* **25**; 1, 1962) pointed out that braking would occur if the gas emitted by a star was magnetically constrained to co-rotate with the star out to distances that were large compared with the stellar radius. Under these circumstances a small amount of mass loss would yield a proportionally much greater loss of angular momentum, simply due to the effective increase in the moment of inertia of the outflowing gas. The efficiency of the braking depends on the strength of the subsurface convection which is itself closely related to the mass of the main-sequence star. This convection is also responsible for the surface activity, such as sunspots, for the corpuscular emission associated with these spots and for part of the general magnetic field of the star.

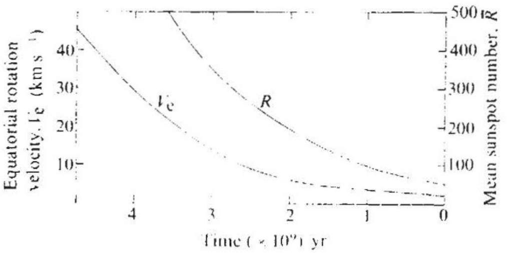
Edward H. Geyer of the Hoher List Observatory, University of Bonn, has studied the braking of solar rotation by magnetic activity and his conclusions have recently been published in *The Moon and the Planets* (**24**, 399; 1981). Geyer compares the Sun with the primary component of the eclipsing binary XY Ursae Majoris (UMa). This is a solar-type star, with similar mass but about 50 times more magnetic activity than the Sun. The binary has an orbital period of 11.5 hours, the components are close and suffer considerable tidal deformation. The XY UMa primary has a spot cycle of about 3.7 years, in comparison with the solar cycle of 11.04 years. At maximum the XY UMa primary has about 20 per cent of its surface covered in spots. XY UMa is spinning quickly — the braking that would lead to a loss of rotational angular momentum is counteracted by a replenishment from the much larger orbital angular momentum of the secondary by a tidal feedback mechanism.

The rotational angular momentum of a star on the lower main sequence determines its spot activity. As magnetic brakes have been applied to the Sun for at least the past 4.6×10^9 years, the Sun must have been spinning faster and showing more magnetic activity in the past. Geyer assumes that the loss of angular momentum takes place at the same heliographical latitudes (5° to 40° north and south) as the maximum spot activity; this maintains the differential rotation. He also assumes that the mass loss is less than a few per cent of the present mass and that the radius has remained constant. He concludes that the angular velocity and the spot activity both decay exponentially, with a mean lifetime of 1.55×10^9 years. Today the mean sunspot number, $\bar{R}$, is 51 ± 35 and the equatorial spin velocity is 2.02 km s^{-1}. The values of these quantities in the past are plotted in Fig. 2.

The effects of these changes are interesting. In the past, the faster spinning Sun would be more oblate, which would perturb the orbital motion of the inner planets. The greatly enhanced sunspot activity must have influenced the evolution of life on the Earth. The increase in solar particle flux and extreme ultraviolet radiation would disturb the terrestrial magnetic field and the original atmosphere; the production rates of carbon-14 and tritium would be higher than they are today; and the atmosphere would be enriched in ^{3}He.

Geyer concludes that evolution of life and its transference from the protecting water to the exposed land were governed by the solar activity and its decay with time. □

Fig. 2 The decay of the solar equatorial rotational velocity, V_{3e}, and the mean sunspot activity, $\bar{R}$, as a function of time. Zero represents the present day.



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