which is not unrealistic) is, for all practical purposes, equivalent to a genuinely non-periodic oscillation.

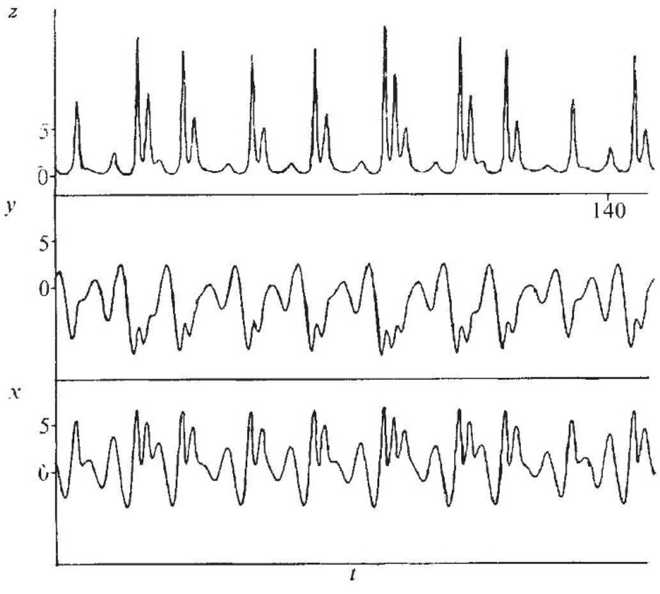
If there is exogenous 'noise' in an apparently chaotic system, the only way to detect an intrinsic chaotic component is through looking at the trajectorial flow in state space. If the flow, although being 'blurred', remains roughly confined to a sub-manifold of detectable shape, and if this shape is one of a few basic types[20] known to be chaos-producing in deterministic conditions, then it is possible that the system is also capable of purely 'endogenous' chaos.

Sometimes a single measured variable (or combination of variables, respectively) suffices for this proof. This is the case in spiral-type chaos[4], for example, if the variable with the largest amplitude is the one that is measured. If subsequent recorded maxima ('amplitudes') are then plotted, each as a function of the respective last, a Li-Yorke-type map is obtained which indicates chaos[12,13]. Other sub-manifolds in state space (like that determining screw-type chaos[7,20]) are not so easily detectable, however. The next step in the present investigation will therefore be to simultaneously record several observables, so that three-dimensional stereoscopic pictures of trajectorial flow in observation space can be obtained.

The simplest screw-type chaotic flow[7] has the temporal behaviour shown in Fig. 2. The time course of the first variable is reminiscent of the flow of Fig. 1. This allows the prediction that typical screw-type chaos can be found in the Zhabotinskii reaction.

The first evidence that the Zhabotinskii reaction is an oscillator of the universal circuit type was provided by the finding of 'double oscillations' in continuous stirred flow conditions[14,15]. At that time, the dynamical implications of this result were still unknown, however. Later, A. T. Winfree (personal communication) suggested that his observation of a 'meandering' core in a non-stirred excitable version of the Z reagent[16] was evidence for chaos. In the meantime, a similar phenomenon has been found in computer studies of a two-variable excitable medium (ref. 17, Rössler and Kahlert, in preparation); while two variables are sufficient for chaos in non-stirred reaction systems[17–19], three variables are necessary in well-stirred systems. The finding that 'non-planarity' of a limit cycle in three- or higher-dimensional state space (so that a 'folded over' regime is possible) is all that is required for chaos[4] immediately suggested that virtually all realistic chemical oscillators should be capable of chaos. Olsen has found evidence for spiral chaos in the horse-radish peroxidase reaction[21,22]. Hudson's group, who had done many experiments on the Zhabotinskii reaction in continuous stirred flow conditions[23], also took a second look and found evidence of chaos[24]. They interpreted their result in terms of the chaotic trajectorial flow described in ref. 4 which corresponds to spiral-type chaos. The present evidence for screw-type chaos (first communicated in ref. 25) does not preclude the possibility of spiral-type chaos in the Zhabotinskii reaction. On the contrary: in abstract systems, the observation of either screw- or spiral-type chaos virtually guarantees that the other type is also possible with slightly changed parameter values[20].

Fig. 2 Time behaviour of the three variables of the simplest screw-type chaotic oscillator. Numerical simulation of the following system of three coupled ordinary differential equations: $\dot{x} = -y - z$, $\dot{y} = x + 0.55y$, $\dot{z} = 2 - 4z + xz$. Initial conditions: $x(0) = y(0) = z(0) = 1$. $t_{end} = 152$. For a stereoscopic picture of the same flow see ref. 20.



We thank Art Winfree for discussions.

Note added in proof: Two-dimensional plots (electrochemical potential against potential of a bromide ion sensitive electrode) obtained in the meantime range from the picture of a fluctuation-triggered monoflop (pseudochaos) to a picture looking like a two-dimensional projection of screw-type chaos.

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Erratum

In the letter 'Surge activity on the Barnes Ice Cap' by G. Holdsworth, *Nature* **269**, p. 588, in paragraph 5 line 32 for computation[10] read computation[13]; in line 40 for balance[10] read balance[13]. In paragraph 8 line 3 for form[3] read form[11].