Our measurements are consistent with the formation of several bipolar charged regions of the cloud consisting of positive charges located above larger negative charges. The charged condition lasted for about an hour before dissipating.

Conductivity measurements near the end of the storm gave

$$4\pi\sigma = 0.90 \text{ s}^{-1} \text{ (e.s.u.)}$$

at the balloon altitude of 27.6 km. This value is in the general range of fair weather values. Using a model of a charged cloud consisting of two unequal charges at different heights, imbedded in an atmosphere with conductivity varying exponentially with height[2-4], we find that typical horizontal and vertical field data may be fitted by an upper charge of 21 C and a lower charge of −35 C, located 50 km from the balloon. With the measured conductivity, the air-earth current due to the storm is 0.12 A.

We conclude that since we have observed strong electric fields above a cloud which did not generate lightning, there must be some process other than lightning which limits charge growth. This work has been supported by a grant from the National Aeronautics and Space Administration.

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Possible new effects in solid state physics

IT has been suggested[1,2] that the electric properties of small enough metallic particles will differ in important ways from those of the bulk material. To test such properties Kreibig and Fragstein[3] made nearly spherical silver particles of given radius R, with 30 Å < $2R$ < 500 Å. This article is concerned with the prediction of new effects which would be observed if particles of similar size consisting of doped and possibly amorphous semiconductors could be produced. The theoretical and experimental background is discussed by Mott and Davis[4].

The simplest example of the effect to be discussed is perhaps activated hopping, for instance in an n-type compensated semiconductor. The disorder can localise the electron states and an activation energy W, provided by interaction with phonons, is necessary for transport. For strongly localised states $W \sim N(E)\,(R_D{}^3)^{-1}$, where $N(E)$ is the density of states and R_D is the average separation between donors. For typical specimens of interest 100 Å < R_D < 400 Å (ref. 4, page 160) and it may therefore be possible to produce particles such that the particle radius $R < R_D$. In this case $W \sim R^{-3}$ independent of donor concentration. This relationship should be even more easily satisfied in the case of weak localisation, in which case $W \sim (\alpha')^3/N(E)$, where the value of α' is such that $(\alpha')^{-1} > R_D$ (ref. 4 page 41). In this again we should obtain $W \sim (N(E)R^3)^{-1}$. A possible experimental test for such effects is the alternating current conductivity (ref. 4, page 51). The hopping distance R_ω, which makes a maximum contribution at frequency ω, is given by $R = (1/2\alpha)\ln(\nu_{ph}/\omega)$ where α is of the order of the inverse Bohr radius and ν_{ph} is a phonon frequency (ref. 4, page 56). The equation $R = {}^1/_2 \propto \ln(\nu_{ph}/\omega)$ can be used to define a critical frequency ω_c for ω, such that for $\omega < \omega_c R < R_\omega$ but for $\omega > \omega_c\ R_\omega < R$. Thus, if $W \sim R^{-3}$ and $\omega > \omega_c$, then $\sigma(\omega) \sim R^6\omega^{0.8}$; whereas for $\omega < \omega_c$, $\sigma(\omega) \sim R^{10}\ \omega$ on the simple theory. This dependence on R and ω could be tested experimentally.

A more dramatic effect seems possible if the current theory of the metal-insulator transition in disordered systems, such as compensated semiconductors, is considered. If the mean square fluctuation of potential is denoted by $U_0{}^2$ and J denotes the bandwidth, then the equation $U_0 > 5J$ is believed to be approximately correct for localisation (ref. 4, page 26). A distance a_E can be defined such that $U_0 = 1/N(E)\,a_E{}^3$ (ref. 4, page 26). For a given energy, and $a_E > R$ this equation becomes $U_0 = 1/N(E)\,R^3$. It follows that for sufficiently small particles the value of U_0 will be large enough for the states to become localised so that a metal-insulator transition will occur. The critical value of R will depend on the energy but will be of the order of R_D at the centre of the band. By the same reasoning it is possible that the region of localised states at the edge of the conduction band in amorphous semiconductors will extend over a greater region of energy in small particles.

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Possible origin of Prandtl's mixing-length theory

PRANDTL's hypothesis[1,2] about turbulent motion in a simple shear layer proposes that the typical values of the fluctuating velocity components in the x and y directions, u and v, are each proportional to $l\delta U/\delta y$ where l is the mixing length (*Mischungsweg*). Prandtl[2,3] says that l "may be considered as the diameter of the masses of fluid moving as a whole in each individual case; or again, as the distance traversed by a mass of this type before it becomes blended in with neighbouring masses. . . ."; and also that l is "somewhat similar, as regards effect, to the mean free path in the kinetic theory of gases". It follows that the Reynolds shear stress $-\rho_{uv}$ is proportional to

$$\rho l^2(\delta U/\delta y)^2$$

and l is defined so that the constant of proportionality is unity. Prandtl, however, described this expression as "only a rough approximation".

Later work (ref. 4, and see ref. 5 for a summary) has shown that the formula for the mixing-length can be derived, using dimensional analysis, by assuming 'local equilibrium' between the turbulence and the mean flow. The only known example of a region of local equilibrium is the wall layer with $l = Ky$: experimentally $K \approx 0.41$. In a self-preserving (self-similar) flow with a single length scale δ, dimensional analysis gives $l/\delta = f(y/\delta)$. These results of dimensional analysis apply to very restricted cases (which were, however, used historically to test the mixing-length theory) and they have no relation to Prandtl's original derivation. In more general cases the quantity l defined by Prandtl has no simple connection with any meaningful scale of the mean flow or the turbulence.

Almost any method of flow visualisation contradicts Prandtl's idea of momentum exchange via lumps of fluid ('*Flüssigkeitsballen*'). Turbulence is an assembly of tangled vortex lines and sheets, dominated by the essentially three-dimensional phenomenon of vortex stretching. If a lump of fluid happens to move away from its former surroundings as an entity, it interacts strongly with those surroundings by the interconnecting vortex lines.

It seems that Prandtl could reconcile his idea with the results of flow visualisation because the predominant method of flow visualisation used in the early years[2,6] of Prandtl's Institute at Göttingen, during the development of the mixing length theory, was the Ahlborn method—the observation of