and the second edition of Nakamoto's book (1970) brought the coverage up to the end of 1967. Now Dr Ross has extended it by a further three years. In accordance with his interest in crystal spectra, the first section of part II concerns compounds which must be regarded as having no discrete molecular vibrating unit. Succeeding sections deal with molecules and ions containing up to eight atoms, or in some cases more. The literature references (1,916 in number) are listed according to first authors, and there is a formula index arranged according to number of atoms and formula type. Researchers in this field will find it useful to have a literature review of this kind in a single book. However, the first four volumes of the *Chemical Society Specialist Reports (The Spectroscopic Properties of Inorganic and Organometallic Compounds)* have also reviewed the literature up to the same date as has Dr Ross. As long as the Chemical Society series continues, little call is likely for further books of the type which Dr Ross has written.

His book is excellently printed and produced. The sole misprint noticed (preface, page xi) raised a smile, for it led to the statement that rotational modes for which the rotations are not free are called "liberations" (*sic*).

L. A. WOODWARD

Nonserial Problems

Nonserial Dynamic Programming. By Umberto Bertele and Francesco Brioschi. Pp. xii+235. (Academic: New York and London, September 1972.) $13.95.

THIS book deals with a combined dynamic programming and graph theoretic approach to optimizing the sum of functions of a set of variables which can take discrete values. The simplest type of problem referred to is

$$\min_X\{f(X)\} = \min_X\{\Sigma_{i\varepsilon T} f_i(X^i)\}$$

where $X=\{x_1, x_2, \ldots x_n\}$, $T=\{1, 2, \ldots n-1\}$, $X^i=\{x_i, x_{i+1}\}$.

This is referred to as the serial unconstrained optimization problem which is well known in the dynamic programming area. It is serial because the $\{x_i\}$ can be completely ordered and the dynamic programming formulation involves only one state variable at each stage. It is, in fact,

$$F_k(x_k) = \min_{x_{k-1}}\{f_k(x_{k-1},x_k) + F_{k-1}(x_{k-1})\} \quad n \geq k \geq 2$$

$$F_1(x_1) = f_1(x_1)$$

with $\min_{x_n}\{F_n(x_n)\}$ required.

The authors do not actually state this equation but the elimination process used is equivalent. First of all x_1 is eliminated by optimizing over x_1 when x_2 is fixed. Then x_2 is eliminated and finally x_n is eliminated.

The main purpose of the book is to generalize the above problem to ones in which X^i does not take the above form. Thus, for example, we might have

$X=\{x_1, x_2, x_3, x_4, x_5\}$;

$X^1=\{x_1, x_2, x_3\}$;

$X^2=\{x_2, x_3, x_4\}$;

$X^3=\{x_3, x_4, x_5\}$

Such a problem is a nonserial problem.

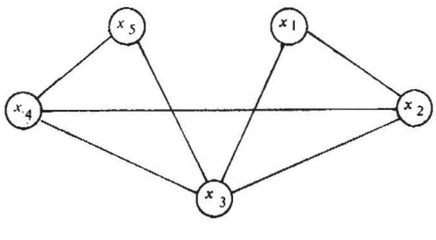
A similar dynamic programming elimination process is used, but in this case, when a variable is eliminated its interactions with other variables have to be considered. Thus we see that: x_1 interacts with x_2 and x_3 because they appear in at least one of the component functions; x_2 interacts with x_1, x_3, x_4 because it appears in component functions containing these variables; similarly x_3 interacts with x_1, x_2, x_4, x_5, while x_4 interacts with x_2, x_3, x_5 and x_5 interacts with x_3, x_4. In solving this problem we can eliminate x_1 first of all giving rise to the optimization problem

$$\min_{x_1}\{f_1(x_1, x_2, x_3)\}$$

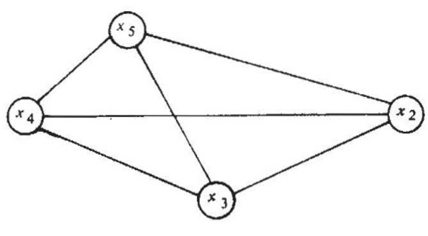
which is a function of x_2 and x_3, say, $h_1(x_2,x_3)$. The original problem is then reduced to a similar optimization problem to the original with three component functions in variables x_2, x_3, x_4, x_5. Having eliminated x_1, if we now eliminate x_2, it interacts with x_3 and x_4 and we find $\min_{x_2}\{h_1(x_2,x_3)+f_2(x_2,x_3,x_4)\}$ which is a function h_2 of x_3 and x_4. The process is repeated. A formal dynamic programming formulation exists, although the authors only do this implicitly in their calculations.

A further extension is to include constraints on some of the variables.

Now the above interaction concept has clear analogies with graphical methods and the following is an interaction graph of the above problem; which is self explanatory.



If we eliminate x_3 first this is equivalent to deleting x_3 and joining up all the others which interact with x_3. Thus we have



The process can be repeated until the graph is reduced to a single vertex.

So far this is fairly routine. The central problem to which the book addresses itself is the one of determining the best elimination sequence from a computational point of view. The amount of computation at each stage (that is, elimination graph) depends both on the degree of a vertex (that is, number of interactions for that vertex for the specific elimination graph) and the number of values each vertex can take. The book considers specific measures of computational effectiveness and makes great use of the concept of dominance. Thus if $d=(d_1, d_2, \ldots d_n)$ and $d'=(d_1', d_2', \ldots d_n')$ are the degrees of the vertices, in the order in which they are eliminated (that is, d_i does not necessarily mean the i^{th} variable), and relative to the elimination graph reached at that stage, then d dominates d' if $d_i \leq d_i'$ for all i and $d_i < d_i'$ for at least one i. Dominance plays an important role in a special class of computational effectiveness functions, $\mathcal{J}$.

The authors then proceed to examine the problem of determining good elimination orderings as a problem in graph theory. The theorems considered are given constructive proofs so that particular properties of specific graphs can be made use of, in a clearly defined manner, in finding such orderings within the class $\mathcal{J}$.

It is possible to formulate the secondary computational problem as an optimal route problem, with its own graphical representation. The authors do this, but do not make use of it, for it is very easy to find the secondary problem more computationally burdensome than the primary problem.

In essence the previous paragraphs capture the general spirit of the book. Additional aspects are special consideration of parametric problems (that is, those in which particular variables are specified to take values in a given range, and the solutions are required relative to each such specification) for which dynamic programming has obvious advantages, and an extension from single variable elimination to block variable elimination, with corresponding treatment of the graph theoretic computational measures as with the single variable elimination process. A limited amount of consideration is given to practical applications.

The book is very well written and amply illustrated. The basic ideas are easy to follow, and, for the less mathematically inclined, an attempt to follow the ideas via the illustrations themselves would be profitable. The full comprehension of the interpretation and proofs of the theorems requires a good background in horticultural graph theory (flowers, bushes, trees, forests and so on) which are, however, defined in the appendix.

D. J. WHITE