LETTERS TO THE EDITOR

PHYSICAL SCIENCES

High Frequency Radar Sea Return and the Phillips Saturation Constant

WE wish to point out a connexion between two independently observed empirical constants: σ^0, the "-23 db" backscattering cross-section of the sea surface to high frequency radar ground waves for a broad range of wind conditions, and the constant $B \approx 0{\cdot}005$ in the Phillips spectrum Bk^{-4} for a saturated sea. This connexion is $\sigma^0 \approx B$.

Barrick and Peake[1-3] have derived the scattering properties of a surface that is "slightly rough", the wave amplitudes being less than the radar wavelength. The sea surface is always slightly rough to hf radar (λ = 10–100 m) and often to vhf (1–10 m). Their procedure matches incident and reflected fields at the statistical surface and uses this to derive the effective surface impedance. The result, when incident field is defined as *total* ground wave field (otherwise the numerical factor is 16π in place of 4π), is

$$\sigma^0 = 4\pi\varkappa^4 \cos^4\theta_i \, \alpha^2 \, [\mathrm{F}(\mathbf{k}^+) + \mathrm{F}(\mathbf{k}^-)]$$

for the average incoherent backscattering cross-section per unit area of sea surface. $\varkappa = 2\pi/\lambda$ is the radar wave number, θ_i is the incident angle relative to the mean surface normal, and $F(\mathbf{k})$ is the directional ocean wave spectrum evaluated at the two wave numbers (Barrick and Peake use $W(\mathbf{k}) = 4F(\mathbf{k})$ in accordance with the traditional Rice normalization).

$$k_x^{\pm} = \pm 2\varkappa \sin\theta_i, \quad k_y^{\pm} = 0$$

determined by the Bragg condition, x being in the direction of the radar beam. Thus the radar scatter (apart from a slight Doppler shift) does not distinguish between the oppositely travelling wave pair $\mathbf{k}\pm$ (as if the sea surface were frozen). The scattering matrix, α, for a perfectly conducting surface and for vertical polarization, both incident and scattered, equals $(1+\sin^2\theta_i)/\cos^2\theta_i$, and for this case

$$\sigma^0 = 4\pi(1+\sin^2\theta_i)^2 \varkappa^4 [F(\mathbf{k}^+) + F(\mathbf{k}^-)] \qquad (1)$$

so that the cross-section is well behaved near glancing incidence, σ^0 approaches $4\pi\varkappa\,[F(\mathbf{k}^+) + F(\mathbf{k}^-)]$ as θ_i approaches 90°, which is the case for so-called ground wave propagation. The result also holds under conditions more general than backscatter.

The ocean wave spectrum is defined so that

$$\overline{\eta^2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(k_x,k_y)\mathrm{d}k_x\mathrm{d}k_y = \int_0^{2\pi}\int_0^{\infty} F(k,\varphi)\mathrm{d}\varphi k\mathrm{d}k = \int_0^{\infty} S(k)k\mathrm{d}k$$

is the mean square elevation. Without loss of generality we can write

$$F(k,\varphi) = S(k)\, g_k(\varphi), \int_0^{2\pi} g_k(\varphi)\, \mathrm{d}\varphi = 1$$

with $g_k(\varphi)$ representing a direction functional. For a directionally isotropic sea $g_k = 1/2\pi$. If the wind comes from φ_0, then g_k is an even function of $\varphi - \varphi_0$ in a growing sea. The form $g_k(\varphi) = 2\pi^{-1}\cos^2(\varphi - \varphi_0)$ for φ between $\varphi_0 \pm 90°$, and zero otherwise, is sometimes used, but there is little justification for this ±45° beam independent of k. The non-directional spectrum function, $S(k) = \int_0^{2\pi} F(k,\varphi)\, \mathrm{d}\varphi$, is relatively easy to measure, by recording the elevation $\eta(t)$ at one point. From many such measurements we find that

$$S(k) = B\,k^{-4}, B = 0{\cdot}005 \qquad (2)$$

provided $k > k_0 = g/U^2$, that is, for waves moving more slowly than the wind speed, U. The form $B\,k^{-4}$ was first suggested by Phillips[4] largely on the basis of dimensional considerations; he attributed the peculiar situation that the power density does not increase with increasing wind (as long as $k > k_0$) to a saturation condition associated with whitecapping. Following these ideas further, Longuet-Higgins[5] was able to derive the observed order of magnitude of the saturation constant B from fundamental principles, starting with the idea that wave energy is lost from the crest of any progressive wave where the local downward acceleration exceeds $\frac{1}{2}g$.

Substitution of the Phillips saturation spectrum into (1) leads to the simple result

$$\sigma^0 = 16\pi\varkappa^4[F(2\varkappa, \varphi_0) + F(2\varkappa, \varphi_0 + \pi)] = \pi B[g_k(\varphi_0) + g_k(\varphi_0 + \pi)]_{\varkappa = 2k} \qquad (3)$$

independent of radar wavelength and wind speed, provided the scattering takes place in the saturated part of the spectrum, $2\varkappa > k_0$ or $U^2 > g\lambda/4\pi$. An upper limit on wind speed is posed by the assumption of a slightly rough surface. The mean square elevation is $\int_{k_0}^{\infty} B\,k^{-4}\,k\,\mathrm{d}k = \frac{1}{2}\,B\,U^4/g^2$, so that $U^2 < g\lambda(\frac{1}{2}B)^{-1/2}$. For 10 m radar (30 MHz), the limits are 3 m/s to 45 m/s which is usually the case.

For an isotropic sea, $g = 1/2\pi$ and $\sigma^0 = B$, or -23 db, in accordance with previous studies (private communication from J. Headrick and A. Peterson). σ^0 will be larger up–down wind and less cross wind, but not enough is known of the directional wave distribution to make meaningful estimates. On the contrary, the radar cross-section should be a good way to obtain such directional information[6,7].

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Observations of an Antarctic Ocean Tabular Iceberg from the Nimbus II Satellite

A TABULAR iceberg 20 nautical miles long was observed by the Nimbus II satellite Advanced Vidicon Camera System (AVCS) from May to August 1966 in an area approximately 350 miles north-east of South Georgia Island. The size and movement of this tabular iceberg suggest several interesting conclusions.

The iceberg was viewed about three times a week with a maximum time between observations of seven days. It was partially obscured by clouds in about thirty-three out of the forty-three sightings. Nevertheless, its location could be determined in all observations and its size could be measured in many pictures contaminated by cloud (Fig. 1).

Measurements of the iceberg were from 70 mm contact prints, and its initial area was calculated to be about 180