LETTERS TO THE EDITOR

ASTRONOMY

Friedman Cosmological Models with both Radiation and Matter

JACOBS[1] has recently given an analytical solution of Einstein's field equations which represents a homogeneous and isotropic universe containing both radiation and dust; the space-like surface (on which the matter is comoving) is flat. Using arguments based on Mach's principle, however, Wheeler[2] argues that space is closed. This implies for a Friedman universe that the curvature is positive. But using statistical arguments and observational data, Chiu[3] argues that the universe can be described by an open Friedman model with negative curvature. This disagreement cannot be resolved at present because the energy density of the universe is not known to sufficient accuracy. Because of this, it seems reasonable to consider all three Friedman cosmological models.

A universe which is spatially homogeneous and isotropic but not necessarily flat can be described by the Robertson–Walker metric[4]

$$ds^2 = -dt^2 + a^2(t)\,[1+(k/4)u^2]^2 \,[du^2 + u^2\,(d\theta^2 + \sin^2\theta d\varphi^2)] \quad (1)$$

where $k = 1$, 0 or -1 correspond, respectively, to positive, zero and negative curvature of the space-like surface t = constant. We also assume that the radiation and dust expand independently and adiabatically and that $(8\pi/3)\ \rho_m = K_m\ a^{-3}$ for the dust and $\rho_R = 3p_R = 3K_r a^{-4}/8\pi$ for the radiation, where K_m and K_r are constants. This choice satisfies the conservation law

$$T^{\mu\nu};\nu = 0 = (\rho a^3)^{\cdot} + p(a^3)^{\cdot}$$

where the dot denotes differentiation with respect to time, ρ is the energy density, and p is the pressure. The only remaining equation to be solved is

$$\dot{a}^2 + k = K_m\ a^{-1} + K_r\ a^{-2}$$

If this equation and the conservation law are satisfied, the other Einstein equation is satisfied automatically.

The three solutions are

$$t - t_0 = (K_r + K_m a - a^2)^{\frac{1}{2}} + (K_m/2)\ \sin^{-1}\,[(K_m - 2a)\,(K^2_m + 4K_r)^{-\frac{1}{2}}]$$

for $k = 1$ (positive curvature);

$$t - t_0 = 2\ (K_m a - 2K_r)\ (K_r + K_m a)^{\frac{1}{2}}/3K_m^2$$

for $k = 0$ (zero curvature);

$$t - t_0 = (K_r + K_m a + a^2)^{\frac{1}{2}} - (K_m/2)\,\log\,[(K_r + K_m a + a^2)^{\frac{1}{2}} + a + (K_m/2)]$$

for $k = -1$ (negative curvature).

The $k = 0$ solution agrees with that of Jacobs[1], but for completeness I have given all three solutions. The requirement that $a = 0$ at $t = 0$ fixes the integration constant t_0.

The Hubble constant $H = \dot{a}/a$ and the deceleration parameter $q = -\ddot{a}/aH^2$ are related to the total energy density and pressure through

$$3_q H^2 = 4\pi(\rho + 3_p)$$

Using the total energy density and the energy density of the radiation, one can (in principle) find the curvature constant k by means of

$$ka^{-2} = (8\pi/3)\ \rho - H^2$$

Because of the uncertainty in the density, however, there is no general agreement on the value of k (refs. 1–3). The situation is complicated still further because the mass density necessary for the binding of our cluster of galaxies is much larger than that observed[5].

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[2] Wheeler, J. A., in *Gravitation and Relativity* (edit. by Chiu and Hoffmann) (Benjamin, New York, 1964).
[3] Chiu, H. Y., *Ann. Phys.*, **43**, 1 (1967).
[4] Robertson, *Ap. J.*, **82**, 284 (1935); Walker, *Proc. Lon. Math. Soc., Ser.* 2, **42**, 90 (1937).
[5] Heckman, O., and Schucking, E., in *Gravitation* (edit. by Witten, L.) (Wiley, New York, 1963) and the references cited there.

Evidence for Lattice Bands in Interstellar Grains

THERE is considerable evidence from the infrared photometry of Johnson[1–3] that the wavelength range 3–10μ is associated with strong emission and absorption of radiation by interstellar grains. Several early type stars which exhibit excess infrared emission have been shown[3] to possess extended circumstellar envelopes. It has been proposed by Stein[4] that the thermal radiation from a size distribution of grains heated to about 300° K could explain this excess radiation. In such a model, which contains several arbitrary parameters, the precise location of the maximum emission at about 10μ must be regarded as rather fortuitous.

It has also been claimed[1] that strong absorption amounting to ~1 mag occurs for some stars in the same wavelength range 3–10μ. To explain these observations, Johnson[1] has suggested that a bimodal size-distribution of grains may be present with peaks at radii of 3μ and 0·3μ. We then require that a sufficient density of grains of 3μ radius are present to give an extinction in the 3–10μ waveband of about 1 mag over a typical distance scale of ~1 kparsec. If a is the radius, Q is the extinction efficiency and s is the density of a grain, the mass extinction coefficient is

$$\varkappa \approx Q\pi a^2 / \frac{4}{3}\,\pi a^3 s \approx 3Q/4as \quad (1)$$

With $Q \approx 0{\cdot}3$, $a \approx 3 \times 10^{-4}$ cm and $s \approx 2{\cdot}5$ g/cm³, we get $\varkappa \approx 300$ cm²/g for the large grains at $\lambda \approx 10\mu$. In order to produce an extinction of about 1 mag we thus require a mass density of about 3×10^{-3} g/cm² projected on the sky. Over a distance of about 1 kparsec, the space density required is about 10^{-24} g/cm³. Although current estimates of the "hidden mass" in the galaxy correspond to a density of this general order, such a high density of heavy elements such as carbon and oxygen cannot be permitted. If the density of these elements were indeed comparable with the neutral hydrogen density, the composition of the surface layers of stars would be drastically different from what is observed. One must therefore look for an alternative explanation of the infrared observations.

We consider here the possibility that grains with impurity induced emission and absorption bands in the 3–10μ waveband may account for the observed effects. Although it does not seem that grains in the general interstellar medium possess strong absorption bands at these wavelengths, the grains in localized HII regions may be considerably different in this respect. Such grains are exposed to a high flux of energetic particles which could produce lattice defects and consequent changes in their optical characteristics[5]. The solid curve of Fig. 1 shows the infrared absorption spectrum of silicon crystals doped with lithium and boron impurities[6]. The dashed curve is the absorption spectrum of silicon crystals irradiated with thermal neutrons[7]. In either case, there are strong absorption bands at frequencies close to