higher-order harmonics are affected first by the masses lying on higher level.

I agree with the second remark—and this is mentioned in my article—saying that the core-undulations cannot be explained by variations of chemical compositions or temperature effects. No difficulty arises, however, if the core is a high-pressure-phase transition of the mantle material only.

In my opinion the third objection is not serious. In the case of the tidal phenomenon also the Earth cannot be considered as rigid. It is still less rigid for a time interval 50,000 times longer.

It is remarkable that there is a correlation, not strong, but positive, between the vertical crustal movements[3] of the coasts and the satellite geoid. This contradicts an explanation by convection currents (Fig. 1).

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PHYSICS

Relativistic (Non-Linear) Oscillator

THE equation of motion of the relativistic oscillator (without damping):

$$\frac{d}{dt}\left(\frac{m_0 \dot{x}}{\sqrt{1-\dot{x}^2/c^2}}\right) + kx = 0 \qquad (1)$$

has been solved in terms of elliptic functions[1]. The result shows the frequency to decrease with the total energy, but does not make explicit how it is related to the amplitude of oscillation. A formula which is more amenable to experimental test is obtained when we approximate the original equation by:

$$\ddot{x} + \omega_0^2 x - \varepsilon x \dot{x}^2 = 0 \qquad (2)$$

$$\varepsilon = \frac{3}{2}\frac{\omega_0^2}{c^2} \text{ and } \omega_0^2 = \frac{k}{m_0} \qquad (2.1)$$

It is found, then, that the frequency shows a red-shift:

$$\frac{\Delta\omega}{\omega} = -\frac{\varepsilon A^2}{8} \qquad (3)$$

where A is the amplitude of oscillation. This result agrees with that derived by another method[2].

The equation of motion of the relativistic oscillator (with damping) is, approximately:

$$\ddot{x} + \omega_0^2 x + b\dot{x} - \varepsilon x \dot{x}^2 = 0 \qquad (4)$$

where $b = \frac{\gamma}{m_0}$ and γ is the damping coefficient. The same red-shift occurs, in first approximation. However, the new frequency is quickly damped out so that this oscillator exhibits the linear frequency ω_0 (ref. 3).

Similarly, the equation of motion of a relativistic oscillator (without and with) damping which is driven by an external force may be written as:

$$\ddot{x} + \omega_0^2 x - \varepsilon x \dot{x}^2 = F \cos \omega_1 t \qquad (5)$$

and

$$\ddot{x} + \omega_0^2 x + b\dot{x} - \varepsilon x \dot{x}^2 = F \cos \omega_1 t \qquad (6)$$

where ω_1 is the driving frequency. The frequency-shift for the undamped, forced oscillator is:

$$\omega_1^2 - \omega_0^2 = -\left(\frac{\varepsilon A^2 \omega_1^2}{4} + \frac{F}{A}\right) \qquad (7)$$

which, if ω_1 is assumed to be close to ω_0, reduces to:

$$\frac{\Delta\omega}{\omega} = -\frac{\varepsilon A^2}{8} - \frac{F}{2\omega^2 A} \qquad (8)$$

For the damped, forced oscillator the shift is:

$$\omega_1^2 - \omega_0^2 = -\frac{\varepsilon A^2 \omega_1^2}{4} - \sqrt{\left(\frac{F}{A}\right)^2 - b^2\omega_1^2} \qquad (9)$$

or, if $\omega_1 \simeq \omega_0$:

$$\frac{\Delta\omega}{\omega} = -\frac{\varepsilon A^2}{8} - \sqrt{\left(\frac{F}{2\omega^2 A}\right)^2 - \frac{b^2}{4\omega^2}} \qquad (10)$$

Thus, in both instances, the frequency-shift is essentially that of the free, undamped oscillator.

Using a light source of high intensity it may be possible to find the red-shift when the light is scattered in a suitable medium. For a ruby laser, having a wave-length λ 6943 Å and an energy flux of 2×10^{32} photons/cm²/sec, a frequency shift $\frac{\Delta\omega}{\omega} \cong 0{\cdot}5 \times 10^{-4}$ is obtained, if the light is scattered in a plasma with frequency $\omega_0 = 10^{12}$ and if the experimental value for the Rayleigh scattering cross-section of 10^{-26} is used[4].

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Transmission of Laser Beams through Various Transparent Rods for Biomedical Applications

THE use of the laser as a surgical tool for the treatment of lesions in tissues other than those of the eye has been reported[1]. Most of these areas treated, however, are those that are readily accessible, making it possible to focus the laser beam directly on the lesion. To apply the laser as an optical knife for surgery, techniques must be developed to direct the laser beam from the laser head to inaccessible areas. This need to change the direction of the laser beam is required not only in the medical field but also in the field of communication.

If the laser beam is needed to impact internal neoplasms and lesions within the body during laser surgery, then it is desirable to use a medium that is safe, flexible, and able to transmit enough energy to be effective without deteriorating. Destruction of portions of pathological lesions has been accomplished by as little as 22 joules/cm² or less, but it may be necessary to transmit at least 110 joules/cm² or more to other areas, especially those tissues that are not pigmented.

Since the laser is light energy, optical instruments have been used to carry or to reflect this light. These include lenses, mirrors, prisms, and rods both of glass and of plastic. When the laser beam was transmitted through a glass lens of optical quality, approximately 10–12 per cent of its energy was lost. Similarly, when it was transmitted through a quartz prism about 8 per cent of its energy was lost with each reflexion and some of the surface was worn away with each shot. These readings were made with a TRG calorimeter and a Kiethley micro-voltmeter. This energy loss is due to absorption, reflexion and refraction of the light.

Mirrors cannot accept high-energy laser impacts without ruining their reflective coating. We have tried aluminium foil as a reflector with the ruby laser; the surface discoloured rapidly at very low-energy densities. Aluminium foil was used as shielding for the protection of tissues about the impact area. Investigations are under way with gold-plated mirrors for the neodymium and ruby laser.