not for more. It was shown by Lagrange that in this case, that is, for two mass-points, Newton's law can be replaced by an equivalent statement whereby the centre of gravity of the two points is at rest and the motion of the two points is described by assuming a potential field with a force inversely proportional to the square of the distance from the centre of gravity. Had Newton enunciated his law in this form, it would, if generalized, have given the Kepler laws for n points in the form in which Kepler expressed them, and this theory would have then become the basis of astronomical investigations, to be refined as observational data demanded. The basic difference from the Newtonian concept lies in the fact that the problems could then be treated in space-time to give a potential field which, in first order, is in accordance with Kepler's laws. Since I can show that a general field given by a Lagrangian function is always the sum of a symmetric (potential) and an anti-symmetric (electromagnetic) field, this concept could be used as the basis for the formulation of a unified field theory.

The difference between such a theory and Einstein's unified theory lies in two points. First, I do not assume that the fields of physics are Riemannian fields, but go back to Hamilton's idea that the only restriction of the Lagrangian function is that it is homogeneous of the first order in the direction cosines. This means, basically, giving up the idea that measurements are independent of the motion of the observer. The very persuasive idea of Einstein, namely, that two light sources in motion with respect to each other can be replaced by a single light source at the instantaneous centre, can scarcely be verified by experiment since two separate sources are not mutually coherent. If the Lagrangian cannot be given by by $(g_{\iota\kappa}\dot{x}_\iota\dot{x}_\kappa)^{1/2}$, then the introduction of non-Euclidean geometry does not serve any purpose. Since the Euler-Lagrange equations in space-time are not only independent of a motion of the chosen origin but are also independent of the choice of variables, it seems to me that experimental evidence cannot enable us to decide between Euclidean and non-Euclidean geometry and that we are therefore free to use the Lagrangian function in Euclidean space-time. The fact that a simple problem like that of a mass-point moving in a potential field cannot be brought into the Riemannian form seems to confirm this deviation from Einstein's ideas.

The second deviation is the idea that, as geometrical optics only forms the skeleton of optical theory along which radiation proceeds until hampered by boundary conditions, I assume that the world lines given by the Lagrangian form only the skeleton for describing the phenomena usually obtained by quantum mechanics.

The great contributions of Einstein, namely, that physics should be considered in a space-time continuum and that the physical laws should be put in a form which is independent of the motion of the observer, are only strengthened by the preceding suggestions. However, the idea that all measurements are independent of the observer is reluctantly abandoned.

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Integral Equation Methods in Elasticity

A NEW method for solving two-dimensional elasticity problems has recently been suggested by N. Louat[1]. It is perhaps worth pointing out that his method falls within the scope of a very general approach to elasticity, which can be briefly described as follows. Regard the body D (assumed homogeneous) as embedded within an infinite elastic medium of the same material, and introduce a distribution of point-forces over its boundary S; since these are acting in an infinite medium, they produce a known displacement field throughout the medium, including D and S; if so any prescribed boundary displacements (or, more indirectly, prescribed boundary tractions) can be achieved by a suitable boundary distribution of point-forces, which hence qualify to generate the relevant elastic fields within D. Mathematically expressed, we formulate the system of three linear functional relations:

$$\sum_{i=1}^{3}\int \sigma_i(\mathbf{q})\ K_{ij}(\mathbf{q}-\mathbf{p})\ dq = U_j(\mathbf{p});\ j = 1,2,3 \qquad (1)$$

where $\mathbf{q},\mathbf{p}$ are vector variables defining points on S, and where dq denotes the surface differential at $\mathbf{q}$; the $\sigma_i(\mathbf{q})$ are the three independent point-force components per unit area at $\mathbf{q}$, and the $U_j(\mathbf{p})$ are the three independent displacement components at $\mathbf{p}$; $K_{ij}(\mathbf{q}-\mathbf{p})$ stands for the j^{th} displacement component at $\mathbf{p}$ produced by $\sigma_i=1$ at $\mathbf{q}$. Given U_j on S, relations (1) become three coupled Fredholm integral equations for the $\sigma_i(\mathbf{q})$, which can then be utilized to generate U_j at any point $\mathbf{P}$ of D simply by writing $\mathbf{P}$ for $\mathbf{p}$ in K_{ij}. By appropriate differentiations and linear combinations of these equations, the problem of given boundary tractions can also be formulated. These considerations hold for bodies of any shape or form, isotropic and anisotropic. Generally speaking, however, not even approximate analytic solutions would be available. Digital computing techniques offer the prospect of achieving effective numerical solutions to problems of technological interest, though the difficulties associated with singular kernels such as K_{ij} and its derivatives should not be underrated.

A close connexion exists between point-forces and dislocations in two-dimensional elastic systems. Thus, the biharmonic equation $\nabla^4\chi=0$ exhibits two independent dislocation solutions $x \log r$, $y \log r$ which involve displacement fields having the same character as those associated with the independent point-force solutions:

$$x\theta + \frac{1-2\nu}{2(1-\nu)}\, y \log r,\ y\theta - \frac{1-2\nu}{2(1-\nu)}\, x \log r$$

where all the symbols have their usual significance. These two formulations are mathematically equivalent when applied to simply-connected domains, but point-force distributions would be required on the internal boundaries of multiply-connected bodies to prevent unwanted dislocations from appearing in the medium. As might be expected, the simplifications afforded by two-dimensional analysis enable several different types of integral equation approach to be developed, some of which prove to be more advantageous than either of those mentioned. In particular, we can write $\chi=r^2\varphi+\psi$, where φ, ψ are harmonic functions in D, and utilize the boundary data to determine the boundary values of φ and ψ, thus reducing the problem to one of potential theory[2].

The idea of a continuous distribution of point-forces over a curved surface seems first to have been exploited by J. D. Eshelby[3] in connexion with elastic inclusion problems. For such problems, however, the distribution is known *ab initio*, thus obviating the necessity of an integral equation formulation.

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