the quantity (Rf/g). The ratio will assume its maximum value when (Rf/g) is a constant over the region of integration. Assigning the value unity to this constant leads immediately to equation (8).

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PHILIP RUDNICK

University of California, San Diego,
Marine Physical Laboratory
of the Scripps Institution of Oceanography,
San Diego 52, California.

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Spherical Probable Error

THE problem of determining an estimate which may reflect light on the accuracy of a weapon system directed against an attacking aircraft has long been recognized. Although efficient methods are well known to mathematicians, the personnel assigned to the execution may not know, and therefore may not utilize, the raw data to form an efficient estimate. Particularly, its importance is to be realized when the data are not complete. This leads me to suggest a new conception, namely, 'spherical probable error'. This is the three-dimensional analogue of the probable error of a single variate. Just as the probable error measures the half-width of the mean-centred interval which includes 50 per cent of the normal probability mass, the spherical probable error measures the radius of the mean-centred sphere which includes 50 per cent of a trivariate normal probability mass. The spherical probable error is defined as:

$$S = c\sigma$$

where $c = 1{\cdot}20645$ and σ is the common unknown standard deviation of the three orthogonal rectangular normal variates with means equal to zero. First two moments of S are given for two cases, namely, when the number of the largest unmeasured observations is: (1) not known, (2) known. Since the spherical probable error is related to σ, the problem of estimating S reduces to that of estimating σ only. The unbiased estimate of σ for the above two cases is given by:

$$\hat{\sigma} = \frac{1}{3}\sum_{i=1}^{3} \sigma_i$$

where σ_i is the maximum likelihood estimate of the standard deviation of the i-th variate as given by Singh[1]. The estimate given in section (2.2) of ref. 1 corresponds to the case (1) and that in section (2.1) to case (2). The asymptotic variance of S under the assumption that σ_i's are uncorrelated is given by:

$$\text{asy. var}\,(\hat{S}) = \frac{c^2}{9}\sum_{i=1}^{3} \text{var}\,(\sigma_i)$$

where:

$$\text{var}\,(\sigma_i) = -\left[E\left(\frac{\partial^2 \log L}{\partial \sigma_i^2}\right)\right]^{-1}_{\sigma_i = \hat{\sigma}}$$

$E\left(-\frac{\partial^2 \log L}{\partial \sigma_i^2}\right)$ will be given by equations (15) and (9) of ref. 1 for case (1) and (2) respectively. It is important and interesting to note that when points of truncation are equidistantly distributed about the mean point of impact, the σ_i will simply be given by s_i, the sample standard deviation, and $E\left(-\frac{\partial^2 \log L}{\partial \sigma_i}\right)_{\sigma_i=\sigma}$ by $\frac{2n}{\hat{\sigma}2}$ for both the cases, n being the size of sample measured.

Theoretical details, other methods for estimating S and tables for facilitating quick solution of S will be submitted for publication later.

NAUNIHAL SINGH

Defence Science Laboratory,
Metcalfe House,
Delhi 6.

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MISCELLANEOUS

A Device for giving a Histogram of Time-Intervals

IN human and animal behaviour studies it is often necessary to measure a large number of time-intervals—for example, stimulus-response times—and to find the mean and variance of the distribution of intervals for the various conditions of the experiment. At present this is either extremely laborious or it involves elaborate and expensive punch tape and computer techniques. The device to be described is designed to give the statistical distribution of time-intervals in a very simple manner. The distribution is built up during the experiment, and the mean and variance may be determined without the use of arithmetic. A photographic record of the distribution may be obtained without the use of a camera.

The histogram is built up by dropping ball-bearings into a row of equally spaced holes in a transparent strip of 'Perspex'. At the beginning of each interval to be included, a ball is made to travel at constant speed across the holes, which thus indicate time-intervals. At the end of each interval a ball is dropped into the hole lying immediately below it. The mechanical problem to be solved is how to accelerate a ball very rapidly from rest, to carry it at constant speed, and finally to drop it exactly when required.

It seemed important to try to limit the moving parts requiring acceleration to the ball itself, and this has been accomplished (Fig. 1). An endless belt, driven at constant speed, lies above the row of holes, and a long narrow electromagnet lies immediately above the belt. At the start of an interval to be included in the distribution, a ball is projected on to the under-side of the belt with a solenoid actuator; it sticks to the underside of the belt, being held by the magnet above it, and it runs along with the belt until the magnet current is cut, at the end of the time-interval. This system is found to work extremely well. It is possible to avoid any slip of the ball on the belt during the period of acceleration by projecting the ball at such an angle that its horizontal velocity equals the velocity of the belt. This angle may be adjusted for the various belt speeds used to give suitable time-scales for the histogram. For situations where the variance is small in relation to the time-interval, a fixed known delay may be introduced, and a suitable belt-speed used so that the display shows the whole of the variance with adequate spread.

A photographic record may be made at any time, by placing sensitive paper behind the display and