LETTERS TO THE EDITORS

COSMOLOGY

Dirac's Cosmology

DIRAC's cosmology was based on the principle "that any two of the very large dimensionless numbers (of the order 10^{39} and 10^{78}), occurring in Nature, are connected by a simple mathematical relation in which the coefficients are of the order of magnitude unity"[1]. From this principle Dirac deduced that the gravitational 'constant' $G \propto (\text{epoch})^{-1}$. In order to describe a cosmological model according to the principles of the general theory of relativity he then changed the units of distance and time so as to make G constant. With the new units the metric can be written:

$$ds^2 = c^2dt^2 - R^2(t)\,(dx^2 + dy^2 + dz^2) \quad (1)$$

with $R(t) = t^{2/3}$. Afterwards I considered this model, and by making certain assumptions, which appeared reasonable, obtained $G \propto \frac{1}{t}$, $t \approx 4 \times 10^9$ years[2]. This gives the values:

$$H \approx 160 \text{ km./sec./}10^6 \text{ parsec.}, \; q = -0{\cdot}5 \quad (2)$$

for Hubble's constant $H\left(= \frac{\dot{R}}{R}\right)$ and the acceleration parameter $q\left(= \frac{R\ddot{R}}{\dot{R}^2}\right)$.

Recently, L. Egyed[3] has proposed a theory of the origin of the solar system which uses these results $\left(G \propto \frac{1}{t}, t \approx 4 \times 10^9 \text{ years}\right)$, and P. Hédervári[4] has used the same results in a theory of the internal structure of the Moon. My main object in this communication is to point out that the assumptions made in these interesting theories are not necessarily associated with the values of H and q given above, but may also give the values:

$$H \approx 80 \text{ km./sec./}10^6 \text{ parsec.}, \; q = -2 \quad (3)$$

from a different description of the Dirac model in Riemannian space-time. It seems advisable to point this out, since the empirical values of H and q may soon be sufficiently accurate to be able to distinguish between (2) and (3).

Rindler[5] shows how cosmological models, including those which are not described according to the principles of the general theory of relativity, may be described in Riemannian space-time, and he gives a metric for the Dirac model of the form (1) with $R(t) = t^{1/3}$. This method of discussing the model is based on the results derived from Dirac's principle, and does not provide a theory of gravitation for the model. Recently, however, I have shown that a theory of gravitation with $G \propto \frac{1}{t}$ may be based on field equations derived from a principle of stationary action[6]. In that theory the characteristic metric was (1) with $R(t) = t^{1/3}$, and the gravitational theory may therefore be associated with the Dirac model. If one assumes $t \approx 4 \times 10^9$ years, this description of the Dirac model gives (3) for H and q.

When one considers the empirical values of H and q it is of interest to observe that McVittie[7] discussed the observational results available up to July 1955 and found values for H of 143–227 km./sec./10^6 parsec. Later, Sandage[8], in studying galaxies in the Virgo cluster, found that Hubble mistook glowing H II regions for brightest stars and, by applying corrections to Hubble's data, found that H can range between 50 and 100 km./sec./10^6 parsec. with a very large uncertainty. De Vaucouleurs[9] suggested that the local group is part of a super cluster of galaxies, and, by making allowance for the velocity of our own system which would occur on account of the differential rotation and expansion of the cluster, he obtained the value $H \approx 140$ km./sec./10^6 parsec. The empirical value of q is also very uncertain at present. Humason, Mayall and Sandage[10] give $q = -3{\cdot}0 \pm 0{\cdot}8$ and Baum[11] finds from photoelectric determinations of red shifts that q has a value in the range $-0{\cdot}5$ to $-1{\cdot}5$.

Thus it would appear that no definite conclusions can be reached from consideration of the empirical results, and that the values of H and q given by either (2) or (3) are within the ranges of these results.

Finally, consider the assumption that $t \approx 4 \times 10^9$ years, in relation to some recent astrophysical calculations. Hoyle[12] has estimated the time required for the evolution of certain stars as $> 10^{10}$ years, but in his calculations he has taken G = constant, whereas in the Dirac model $G \propto \frac{1}{t}$. Hoyle's figures are not, therefore, strictly comparable with the assumption that the age of the universe $\approx 4 \times 10^9$ years. This age is, however, too small to allow time for the synthesis of the heavy isotopes of uranium in a star ($\approx 6{\cdot}6 \times 10^9$ years), by the process described by Burbidge, Burbidge, Fowler and Hoyle[13].

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