LETTERS TO THE EDITORS

SPACE SCIENCE

Satellite Orbit Perturbations in Vector Form

THE first-order perturbation theory of satellite orbits can be formulated simply and elegantly in terms of constants of the unperturbed motion using vector methods. For let the equation of motion be:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} + \mathbf{F} \qquad (1)$$

where $\mu = GM$ (M = mass of the earth), and $\mathbf{F}$ is a general perturbing force (per unit mass). The energy E and angular momentum $\mathbf{h}$ (both per unit mass) defined as:

$$E = \frac{1}{2}\dot{\mathbf{r}}^2 - \frac{\mu}{r}; \quad \mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}} \qquad (2)$$

are constants of the unperturbed motion. Hamilton's integral, which may be written in the form[1]:

$$\mathbf{k} = \frac{\mu}{r}\mathbf{r} + \mathbf{h} \times \dot{\mathbf{r}} \qquad (3)$$

provides a further constant of the unperturbed motion under an inverse square law force. Taking scalar products with $\mathbf{h}$ and $\mathbf{r}$, $\mathbf{k}$ is seen to be a vector of length μe lying along the major axis in the direction of apogee. The orbit is over-determined by E, $\mathbf{h}$ and $\mathbf{k}$ which give seven scalar equations.

In the presence of the perturbing force, E, $\mathbf{h}$ and $\mathbf{k}$ are no longer constant, but from (1) satisfy:

$$\begin{aligned} \dot{E} &= \dot{\mathbf{r}} \cdot \mathbf{F} \\ \dot{\mathbf{h}} &= \mathbf{r} \times \mathbf{F} \\ \dot{\mathbf{k}} &= (\mathbf{h} \times \mathbf{F}) + (\mathbf{r} \times \mathbf{F}) \times \dot{\mathbf{r}} \end{aligned} \qquad (4)$$

When $\mathbf{r}$, $\dot{\mathbf{r}}$ and $\mathbf{h}$ are taken to refer to the unperturbed orbit, these are the equations of first order in the perturbing force.

It is best to discard two components of $\dot{\mathbf{k}}$ since this has the most complicated equation. Then $\dot{E}$ gives the change in the semi-major axis ($E = -\mu/2a$), $\dot{\mathbf{h}}$ the rotation of the plane of the orbit and the change in eccentricity [$h^2 = \mu a(1-e^2)$], while the rotation of the major axis in the plane is given by the component of $\dot{\mathbf{k}}$ parallel to the minor axis.

The quantities E, $\mathbf{h}$ and $\mathbf{k}$ will give the osculating elements at any time. However, for most purposes the interesting effects are those of period longer than the satellite in its orbit. These are given by the mean rates of change obtained by integrating (4) over a complete orbit, using either the true or the eccentric anomaly, as appropriate, as the independent variable.

The results can often be expressed partly in vector form. For the simple case of a constant force $\mathbf{F}$, corresponding, for example, to radiation pressure on a uniform sphere the orbit of which is entirely in sunlight, the effects are given by:

$$E = 0, \; \dot{\mathbf{h}} = \frac{3}{2}\frac{a}{\mu}\mathbf{k} \times \mathbf{F}, \; \dot{\mathbf{k}} = \frac{3}{2}\mathbf{h} \times \mathbf{F} \qquad (5)$$

This method has been applied to solar and lunar perturbations and to the effect of radiation pressure when part of the orbit is in darkness. It has also been applied to the second harmonic term in the Earth's gravitational field giving results in agreement with previous work[2].

A treatment essentially in terms of E, $\mathbf{h}$ and $\mathbf{k}$ appears to have been proposed first by Herrick[3], and Musen[4] has applied it to the case of a constant perturbing force giving effectively the results of (5) above.

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[2] King-Hele, D. G., *Proc. Roy. Soc.*, A, **247**, 49 (1958).
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The Cosmic-Ray Observations of December 4, 1957

A RECENT communication from Legrand and Helary[1] reports an increase in the total component of cosmic radiation at Paris on December 4, 1957. The increase amounted to about 18 per cent and extended over three hours with the maximum occurring at 2245 U.T. A small increase in the nucleonic component at Thule preceded the Paris event by about three hours. Optical and radio data indicate a flare of importance 1 on the western hemisphere of the Sun at 1657 U.T. (ref. 2).

Intensity enhancements at the surface of the Earth are comparatively rare, only eleven cases having been observed. However, these cosmic ray flare events appear to have certain well-defined properties in common. These include correlation in time with a visible or inferred solar flare, a steep integral rigidity spectrum and approximate simultaneity of occurrence at all points on the Earth's surface accessible to the radiation. The steep rigidity spectrum is indicated by the relative rarity of events seen in ground-level neutron monitors as compared to detectors flown in balloons and the small magnitude (or absence) of the increases measured in the meson component compared to the nucleonic enhancements.

Collins and Jelly[3] have reported that ionospheric conditions were normal throughout the period in which the increase in cosmic radiation occurred and have also pointed out that solar activity was low. Since an undisturbed ionosphere at the time of a cosmic-ray flare is an unlikely event, it was thought worth while to examine the intensity of cosmic radiation monitored at the Sulphur Mountain Laboratory at this time. The Laboratory is on a mountain near Banff, Alberta, Canada (lat. 51·2° N., long. 115·6° W.), at 7,500 ft., and houses both neutron and meson monitors. The neutron monitor, consisting of three boron trifluoride counters in a paraffin – lead pile, has a counting-rate of 100,000/hr., and a cubical Geiger counter telescope containing 10 cm. lead