LETTERS TO THE EDITORS

MATHEMATICS

Analysis of Accelerated Motion in the Theory of Relativity

Conventional treatments of accelerated motion in the theory of relativity have led to certain difficulties of interpretation. Thus, Crampin, McCrea and McNally[1] mention the lack of uniformity in the correspondence of events as depicted by the transformation of Born and Biem. Again, Donahue and Leffert[2] and Moller[3] discuss certain reversals in the apparent gravitational field of an accelerated body. I have found[4] that these difficulties may be avoided by simpler analysis based on the use of restricted conformal transformations. In the conformal theory the velocity of light remains constant even for experimenters in accelerated motion.

The problem considered is that of rectilinear motion with a variable velocity v. I introduce two co-ordinate systems, A, (x,t) and B, (x',t'). The motion takes place along the x or x' axis.

The correspondence between the xt and $x't'$ systems may be expressed quite simply by the transformation:

$$\begin{aligned} x' + t' &= F(x + t) \\ x' - t' &= G(x - t) \end{aligned} \qquad (1)$$

The velocity is given by:

$$v = \frac{g - f}{g + f} \qquad (2)$$

Here f and g are the derivatives of F and G with respect to their arguments $x \pm t$. I now suppose that A, (x,t) is an inertial system, and in order to satisfy the relation of equivalent scale in the vicinity of B, I apply the boundary condition:

$$fg = 1 \text{ along } x' = 0 \qquad (3)$$

If the motion of B is given then this relation, together with equation (2), is sufficient to determine the functions F and G. As determined in this way, this transformation becomes tangent to instantaneous Lorentz transformations all along the path of B and in the case of uniform velocity reduces to the Lorentz transformation everywhere.

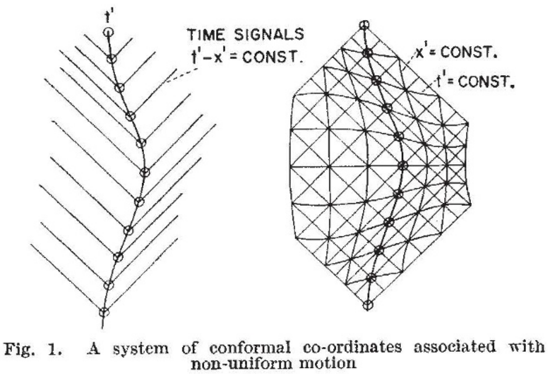

Fig. 1. A system of conformal co-ordinates associated with non-uniform motion

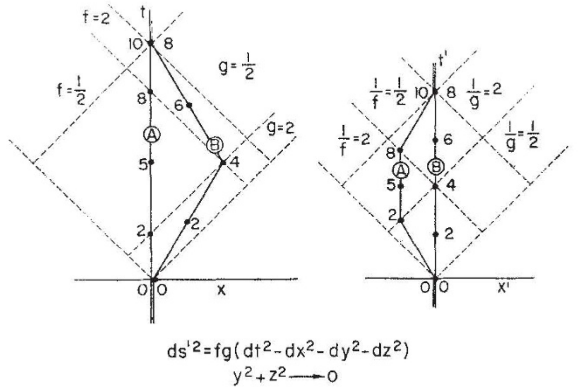

Fig. 2. Example showing application of restricted conformal transformations

Fig. 1 shows how such $x't'$ co-ordinates may be constructed graphically with the aid of periodic time signals originating in the B system. Such signals generate a family of outgoing waves, represented by the characteristic lines $t \pm x$ = constant. If these lines are now extended backward, a corresponding family of incoming signals will be represented. Intersections of the characteristic lines may then be identified with the events of synchronization of the various clocks of the B system.

Extension of the theory of relativity by conformal transformations in four dimensions was considered many years ago by Bateman[5]. It seems that the group $C4$ admits only restricted motions, and of these the Lorentz transformation alone maintains equality in the scale relation. Therefore, we do not speak of a conformal transformation of the whole space. However, by restricting attention to a narrow cylindrical region around the x-axis, conformal mappings can be employed locally, so that:

$$ds'^2 = fg(dt^2 - dx^2 - dy^2 - dz^2) \qquad (4)$$

for $y^2 + z^2 \to 0$.

Fig. 2 illustrates a simple example of the type discussed in connexion with the clock paradox. At $t = 0$, B starts away from A at the velocity 3/5. At $t = 5$, B reverses its motion and returns. Values of f and g in various regions are indicated between characteristic signal lines. The condition of local scale equivalence, $fg = 1$, results in a 20 per cent reduction of the elapsed time along the path of B. In addition to the time discrepancy, there appears also a discrepancy in the relative spatial displacements.

Robert T. Jones
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[2] Leffert, C. B., and Donahue, T. M., *Amer. J. Phys.*, **26**, No. 8 (1958).
[3] Moller, C., *Amer. J. Phys.*, **27**, No. 7 (1959).
[4] Jones, Robert T., "Extending the Lorentz Transformation to Motion with Variable Velocity", *NASA* Memo 7-9-59*A* (1959).
[5] Bateman, H., *Proc. Lond. Math. Soc.*, ii, **8**, 223 (1910).