THE STATISTICS OF CHANGE

An Introduction to Stochastic Processes, with Special Reference to Methods and Applications

By Prof. M. S. Bartlett. Pp. xiv+312. (Cambridge: At the University Press, 1955.) 35*s*. net.

THE theory of stochastic processes is the mathematics of systems which change in accordance with probability laws. There are many applications —for example, to the study of epidemics, cosmic ray showers, irregularities in textile processing, queueing and storage problems, the growth and mutation of bacterial populations, and communication theory. Prof. M. S. Bartlett has made distinguished contributions to many branches of the theory, and now he has produced this masterly introduction to the whole field. He writes as a statistician and applied mathematician and is therefore thoroughly justified in avoiding detailed discussion of various matters, which, while of considerable mathematical interest, can easily divert attention from the aspects of most importance in applications. The result is that he has been able to cover concisely a remarkable range of methods and problems and that, while the book is highly mathematical, the mathematics is always directed towards obtaining a working answer to the problems under discussion. The pure mathematical aspects of stochastic processes have been dealt with in a recent book by J. L. Doob; the point of view of the two books is entirely different, and Prof. Bartlett's is the only one at present available for the more applied worker in this field.

After a preliminary chapter devoted to basic definitions and to recalling some results in probability theory, there is a chapter on random walks and Markov chains occurring in 'discrete time'; one of the applications described is to the statistical mechanics of nearest-neighbour systems. There is next a chapter on the corresponding processes in continuous time. Particularly noteworthy here is the discussion of passage and recurrence time problems, based on an elegant symbolism for determining the Laplace transforms, or probability generating functions, of the distributions of the times taken for specified types of transition to occur. This section of the book is completed by a long chapter on miscellaneous applications. The topics discussed here include population growth and epidemic models.

The next group of chapters deals with stationary processes, that is, systems which are in statistical equilibrium. After a preliminary chapter, there is an account of the spectral function and of the generalized harmonic analysis of stationary processes. This is followed by a discussion of recurrence times and, among other things, a brief account of applications to the theory of turbulence. The third chapter of this group deals with prediction and communication theory.

The final two chapters discuss problems of statistical inference connected with stochastic processes, the last chapter being about the correlation and spectral analysis of stationary time-series. This gathers together in a valuable way recent work on this subject. The emphasis is on situations where the object of the statistical analysis is to examine the structure of the observed process. It would have been interesting to have had also some discussion of other applications, for example, to problems of sampling, where the precise structure of the process is not the point at issue. It might also have been helpful to have had some mention of processes which, while stationary, show appreciably greater long-term fluctuations than processes usually considered in the theory. An example is when the spectral density behaves like $\omega^{-\alpha}$, $0 < \alpha < 1$ as ω tends to zero. This seems to happen in some applications, for example, in certain textile problems where there are sources of random irregularities with very long wave-lengths.

The presentation is open to minor criticism. In detail the writing, though clear, is very concise, and the book is not easy reading. Some familiarity seems desirable with the elementary parts of the subject (such as those given in the book by W. Feller); results for simple processes are almost always obtained as special cases of general formulæ, rather than used for preliminary illustration of the ideas involved, as would, perhaps, have been advisable in an introductory work. Also, the assumptions for special applications are occasionally set out only in symbolic form. This will cause no difficulty to anyone reading the book systematically, but is hard on the reader who wishes just to find out what is assumed in the discussion of some particular application of interest. However, the book is a characteristically original, impressive and important contribution to the subject, and deserves to be widely read. D. R. Cox

MEASUREMENT AND AUTOMATIC CONTROL

Instrument Engineering

(McGraw-Hill Publications in Aeronautical Science.) By Charles Stark Draper, Prof. Walter McKay and Prof. Sidney Lees. Vol. 1: Methods for Describing the Situations of Instrument Engineering. Pp. xvi+269; 51*s*. Vol. 2: Methods for Associating Mathematical Solutions with Common Forms. Pp. xxviii+827; 120*s*.
(London: McGraw-Hill Publishing Company, Ltd., 1952, 1953.)

THESE volumes are out of the ordinary, both in magnitude and in presentation of the subject-matter. They are the first two of three volumes—the third is not yet available—which combine the functions of introductory text-book and comprehensive reference book of mathematical techniques relating to "the art and science by which the properties of matter and the laws of nature are made useful to man in the observation and control of physical processes".

The authors are members of the Aeronautical Engineering Department of the Massachusetts Institute of Technology, and these books are the outcome of twenty years experience in research and postgraduate teaching in the field of aircraft instrumentation and control. This background has largely influenced the presentation of the subject-matter, and made possible the accumulation of the very large number of most useful results of mathematical analysis which are now made more widely available.

The need to find a common language and symbolism for mathematicians, physicists and a diversity of engineers—aeronautical, electrical and mechanical—in a field to which all have combined to contribute has led the authors to devise an elaborate, but most carefully defined, nomenclature and symbolism, which are used throughout the volumes. This leads, however, to such distressingly cumbersome statements as: ". . . it appears that, from the standpoint