



Fig. 2. A, in hydrogen; B, in oxygen

current at 60 c./s., and measuring the induced 10 c./s. current. This gave a value of 1.77×10^5 cm.³/coulomb for the coefficient of an electron-excess semi-conductor at 442°. The conductivity of this specimen was 1.1×10^{-5} ohm⁻¹ cm.⁻¹. We are greatly indebted to Mr. D. A. H. Brown for his interest and participation in these measurements.

These preliminary results indicate that such methods for examining mixed catalysts offer considerable promise, but that great care is essential in the preparation and handling of specimens. In the present series, the semi-conducting properties begin to be measurable in the same temperature-range as the activity for dehydrogenation of heptane and for chemisorption of hydrogen on the clean catalyst. It therefore seems possible that the centres concerned in chemisorption are the same as those responsible for electrical conductivity. If this were so, the activation energies for the two processes should be closely related, and measurements are being continued to investigate this point.

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A Resistance-Network Analogue Method for solving Plane Stress Problems

PLANE stress problems can be described by the Airy stress function χ , which satisfies the biharmonic equation

$$\nabla^4 \chi = 0, \quad (1)$$

the stress components being the second derivatives of this function. A resistance network method has been developed which solves the biharmonic equation.

As has been shown by several authors¹⁻³, a network of interconnected resistances solves the second-order partial differential equation

$$\nabla^2 \chi(x,y) = \Phi(x,y), \quad (2)$$

the function $\Phi(x,y)$ representing the current fed into the network node (x,y) . Now suppose that $\Phi(x,y)$ is the solution of a second-order differential equation,

$$\nabla^2 \Phi(x,y) = f(x,y), \quad (3)$$

where $f(x,y)$ is a known function of the co-ordinates x and y . Then obviously

$$\nabla^4 \chi(x,y) = f(x,y). \quad (4)$$

To establish the solution of equation (4), one need only connect two identical resistance networks in cascade by joining corresponding nodes in the two networks by (relatively) high-value series resistances, and feed in currents at the 'upper' network corresponding to $f(x,y)$, while applying the boundary conditions appropriate to $\Phi(x,y)$ at the 'upper' network. The voltages $\Phi(x,y)$ then appearing in the 'upper' network, and representing the 'solution' of equation (3), constitute the sources for the currents fed into the 'lower' network, the currents being proportional to these voltages. Hence, equation (4) is satisfied by this network arrangement. This principle might be extended to the solution of partial differential equations of higher even orders, but practical difficulties arise through the potential division in the network cascades, and the ensuing low signal-level in the 'lowest' network. This particular difficulty makes itself already felt in the network cascade to solve the biharmonic equations (1) or (4), because relatively high accuracy in the determination of the χ function (a few parts in 10,000) is required, in order that the stress components can be obtained with an accuracy of 1-2 per cent by numerical differentiation of the measured $\chi(x,y)$ values. However, this accuracy has now been achieved in an experimental apparatus set up to test the practicability of the method.

Another difficulty is that usually unfavourable boundary conditions are specified, for example, χ_B and $(\partial\chi/\partial n)_B$, which correspond to the case of 'ill-conditioning' in numerical methods of solving equation (1)⁴. This has been overcome by setting up one of the boundary conditions, for example, $(\partial\chi/\partial n)_B$, and displaying (on a cathode ray oscillograph screen) the other one, χ_B , simultaneously for all boundary points. The boundary values of the Φ function (at the 'upper' network) are then adjusted until the observed χ_B values have their prescribed values within specified limits. This mode of operation is, to some extent, the experimental counterpart of Southwell's relaxation method⁵, but the networks are 'self-relaxing' in their interior⁶, and the error signals along the boundaries are all simultaneously in view. Hence, although the problem is 'ill-conditioned', a solution of sufficiently high accuracy can usually be established in a relatively short time; for example, present experience has shown that in a typical case, with a total of about 250 mesh points, and twenty-five displayed boundary points (the other boundary points having fixed boundary values), the solution can be established in less than half an hour to within 3 parts in 10,000. Of course, the extraction of the full information and the evaluation of the stresses at all mesh points takes much longer.

A more detailed account of the method and its experimental realization will be published elsewhere.

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