and $A(\theta,\varphi\,;\,x)$ can be expressed as

$$\frac{1}{2}\,\frac{\cos(\theta-\varphi)x}{\theta-\varphi}\,[\psi(1+\theta)\;-\;\psi(1+\varphi)]\;+$$

$$\frac{1}{2}\,\frac{\cos(\theta-\varphi)x}{\theta-\varphi}\,[c(\theta,2x)\;-\;c(\varphi,2x)]\;+$$

$$\frac{1}{2}\,\frac{\sin(\theta-\varphi)x}{\theta-\varphi}\,[s(\theta,2x)\;+\;s(\varphi,2x)],$$

where

$$\psi(z)=\frac{\Gamma'(z)}{\Gamma(z)},\;c(\theta,x)=\sum_{n=1}^{\infty}\frac{\cos(n+\theta)x}{n+\theta},$$

$$s(\theta,x)=\sum_{n=1}^{\infty}\frac{\sin(n+\theta)x}{n+\theta}.$$

The transformations for $c(\theta,x)$ and $s(\theta,x)$ were obtained by G. H. Hardy[4].

Putting $\theta=\varphi$ and taking the sum of $A(\theta,\theta\,;\,x)$ and $A(-\theta,-\theta\,;\,x)$, one obtains the result (Krishnan, *loc. cit.*):

$$\sum_{n=-\infty}^{\infty}\frac{\sin^2(nx+\alpha)}{(nx+\alpha)^2}=\frac{\pi}{x}.$$

Putting $\theta=-\varphi=\alpha m$ ($\alpha>1$, $m=1,2,3\ldots$) and $x=\pi/\alpha$, one obtains (Goddard, *loc. cit.*):

$$\sum_{n=1}^{\infty}\frac{\sin^2(n\pi/\alpha)}{n^2-\alpha^2m^2}=0.$$

The corresponding series involving the cosines, namely,

$$\sum_{n=1}^{\infty}\frac{\cos(n+\theta)x}{n+\theta}\cdot\frac{\cos(n+\varphi)x}{n+\varphi}=B(\theta,\varphi\,;\,x),\text{ say,}$$

can also be treated in the same manner.

The last series can be used to obtain the result

$$\sum_{n=-\infty}^{\infty}\frac{\cos^2(nx+\alpha)}{(nx+\alpha)^2}=\left(\frac{\pi}{x}\right)^2\left[\operatorname{cosec}^2\left(\frac{\pi x}{\alpha}\right)-\frac{x}{\pi}\right].$$

The details of the calculations and certain other results will be published elsewhere.

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[1] Krishnan, K. S., *Proc. Roy. Soc.*, A, **192**, 181 (1947).
[2] Krishnan, K. S., *J. Ind. Math. Soc.*, **12**, 79 (1948).
[3] Goddard, L. S., *Proc. Camb. Phil. Soc.*, **41**, 148 (1945).
[4] Hardy, G. H., see Bromwich, T. J. I'A., "An Introduction to the Theory of Infinite Series", 392 (Macmillan, 1926).

Multiple Meson Production

It has recently been reported by Daniel *et al.*[1] that in high-energy collisions between nucleons, besides π-mesons, some heavier mesons of about 1,300 m_e mass are also created. The purpose of the present note is to estimate the theoretical probability of emission of a heavy meson in a collision process, using the statistical theory developed by Fermi[2]. The collision is studied in the centre-of-mass system of the particles and it is assumed that the total energy W (W is the energy in the centre-of-mass system) is suddenly released in a small volume,

$$\Omega=\frac{4\pi}{3}(\hbar/\mu c)^3 2Mc^2/W,$$

surrounding the nucleons (M = mass of the nucleon, μ = mass of π-meson). One then calculates statiscally the probability that a certain number of mesons are created.

On a suitable modification of Fermi's results, we find that the probability, $P(s,n)$, of two nucleons, s heavy mesons (of mass K) and n π-mesons being present in the final state of collision, is given by:

$$P(s,n)=\frac{8\cdot5}{(2+s)^{3/2}}\,\frac{(29\cdot3/\omega)^{s/2}}{\{(6n+3s+1)/2\}\,!}$$

$$\left\{\frac{6\cdot31}{\omega^{1/3}}\left(\omega-2-\frac{2}{3}\,s\right)\right\}^{3n+\frac{3}{2}s+\frac{1}{2}}\exp\left\{-\frac{6\cdot31}{\omega^{1/3}}\,(\omega-2)\right\}$$

where $\omega=W/Mc^2$ and we have taken $K/M=2/3$; $\mu/M=0\cdot15$. In deriving the above result the nucleons and the heavy mesons have been treated in the non-relativistic approximation, whereas π-mesons have been treated in the extreme relativistic approximation. Further, the conservation of angular momentum has not been considered.

The table gives the results obtained from the above formula.

ω	Energy of primary particle in laboratory system	Average number of π-mesons produced when no heavy meson is created	One heavy meson is created		Ratio of the number of heavy mesons to π-mesons created in a collision
			Probability of the event*	Average number of π-mesons produced	
3	$3\cdot5\ Mc^2$	$1\cdot2$	7		$\sim$ 1/17
4	$7\cdot0$	$2\cdot5$	9	1	$\sim$ 1/29
5	$11\cdot5$	$3\cdot5$	11	$2\cdot2$	$\sim$ 1/34
6	$16\cdot9$	$4\cdot4$	12	$2\cdot3$	$\sim$ 1/39

* The probability of the event when no heavy meson is produced is taken to be 100.

It will be noticed that, for collisions involving primary energies up to about 20 Mc^2, the possibility of a heavy meson being created is small. (The relative probability of two or more heavy mesons being created is very small and does not alter the conclusions drawn. For very high energies the above formula breaks down.) The slightly higher ratio of the number of heavy mesons to that of π-mesons, near the threshold energy for the creation of a heavy meson, will also be observed. This is due to the fact that, according to this theory, the probability of creation of a heavy meson varies more slowly with increasing energy than does the average number of π-mesons created.

In the extremely high energy region, where the heavy mesons and the nucleons have also to be treated in the extreme relativistic approximation, it follows from thermodynamic considerations[2] that the number of heavy mesons produced will be comparable to the average number of π-mesons, the ratio being governed mainly by the ratio of their respective statistical weight factors.

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[1] Daniel, R. R., *et al.*, *Phil. Mag.*, **43**, 753 (1952).
[2] Fermi, E., *Prog. Theor. Phys.*, **5**, 570 (1950).