this source, in the region of the iso-ionic point.

Other factors not included in the above theory are: (1) the electroviscous effect of Smoluchowski<sup>6</sup>; (2) changes in the pK's of the basic and acidic amino-acid residues along the gelatin molecule with changing ionic environment.

It is considered unlikely that (1) would lead to changes of more than a few per cent in the reduced viscosity, although an accurate estimation cannot be made as no satisfactory theoretical treatment is available. In a more detailed analysis of the curves, it is possible that both the displacement of the peaks to more extreme values of pH, and the more rounded maxima, when neutral salts are present, are associated with (2).

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Pouradier, J., and Venet, A. M., J. Chim. Phys., 47, 11 (1950). <sup>1</sup> Method of de-ionization: Janus, J. W., Kenchington, A. W., and Ward, A. G., Research, 4, 247 (1951).
<sup>2</sup> Simha, R., J. Phys. Chem., 44, 25 (1940).
<sup>4</sup> Hitchcock, D. I., J. Gen. Physiol., 15, 190 (1931).
<sup>5</sup> Katchalsky, A., J. Polymer Sci., 7, 393 (1951).
<sup>5</sup> Smoluchowski, Koll. Z., 18, 190 (1916).

## Strength of Reinforced Concrete under the Action of Combined Stresses, and the Representation of the Criterion of Failure by a Space Model

In concrete, as in most other brittle materials, two distinct types of failure can be distinguished. A crushing failure is always accompanied by the formation of debris, and is generally caused by diagonal shear due to compression. A cleavage failure is easily recognized by the clean appearance of the fracture; if the concrete is suitably reinforced, failure in direct tension due to bending can be prevented, and a cleavage fracture is therefore normally the result of diagonal tension due to shear.

Leon<sup>1</sup> has proposed Mohr's theory with a single enveloping curve as the criterion of failure for concrete. It is, however, difficult to reconcile the experimental data with this conception without assuming a very sharp change in the curvature of the envelope in the tensile region. There is, moreover, no apparent reason why a single criterion should account for two entirely different types of failure.



Fig. 1. Space models representing theory of failure for reinforced concrete. (*xx, yy, zz* represent the three principal stresses, the positive sign denoting tension)



Fig. 2. Bending moments and twisting moments at the breakdown of elastic action. Experimental results

Richart and Brandtzaeg<sup>2</sup> and Balmer<sup>3</sup> have examined the crushing failure of concrete in triaxial compression. Although the envelope of the Mohr circles becomes curved for high triaxial stresses, a straight line inclined to the direct stress axis at the angle of internal friction of the aggregate gives a reasonable approximation in the lower stress range, that is, the range which is of particular interest in the design of engineering structures. The substitution of Coulomb's internal friction theory for the generalized Mohr theory makes it possible to express the criterion for the crushing failure of concrete in simple mathematical terms.

The experiments of Fisher<sup>4</sup> and Nylander<sup>5</sup> in combined bending and torsion generally support the use of Rankine's maximum stress theory as the condition for cleavage failure.

The two theories may be represented by the space models in Fig. 1 (a) and (b). The combined model in Fig. 1 (c) represents the proposed criterion of failure for reinforced concrete under the action of combined stresses.

I have checked this criterion by experiments in combined bending and torsion. Reinforced concrete fails in torsion with a cleavage fracture perpendicular to the direction of the principal tensile stress. In bending, tension failure is prevented, and the beam therefore fails in diagonal shear due to compression. The bending moments and twisting moments of the test specimens at the breakdown of elastic action are shown in Fig. 2, together with the limiting values represented by the surfaces of the model in Fig. 1 (c). Primary torsion failure corresponds to the cleavage (Rankine) surfaces of the model, primary bending failure to the crushing (Coulomb) surfaces. An interesting result is the increase in torsional strength resulting from the addition of bending.

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- <sup>1</sup> Leon, A., Der Bauingenieur, 15, 318 (1934).
- <sup>2</sup> Richart, F. E., Brandtzaeg, A., and Brown, R. L., Univ. Illinois Eng. Exp. Sta. Bull. No. 185 (1928).
   <sup>8</sup> Balmer, G. G., U.S. Dept. Int., Bur. Reclamation, Struct. Res. Lab. Rep. No. SP.23 (1949).
- <sup>4</sup> Fisher, D., Ph.D. (Civ. Eng.) thesis, Univ. London (1950).
- <sup>5</sup> Nylander, H.. Statens Kommitté för Bygnadsforsking, Medd., No. 3 (Stockholm, 1945).