

than their fair share of attention. It is proposed that "biotic potential" be corrected by an "internal adjustment factor" to allow for changes in reproductive-rates as numbers increase; this figure minus "environmental resistance" is stated to equal the population (apparently not its rate of change).

It can certainly be argued that these and similar abstract terms are useful in describing population processes that we cannot yet understand. On the other hand, there is a danger that the label may be confused with what it describes, and lead to an inflexibility of thought not warranted by the evidence. For example, the following views are presented: an environment has a certain carrying capacity such that animals in excess of it will be lost; the carrying capacity may be increased by improving the environment up to a saturation point which is "a function of the animal psychological pattern, rather than of the land"; predators remove individuals in excess of the carrying capacity but otherwise prey chiefly on the weaker members; predators are effective in controlling certain rodents (but not the grey and fox squirrels, which are game animals).

Now these are controversial matters which should be treated with extreme caution. The present book is unsatisfactory in this respect, particularly since authorities are not quoted in the text. It is true that a policy based on these hypotheses has the attraction that it may lessen the slaughter of predators and encourage 'common-sense' procedures such as providing food in winter and planting cover. But the sad fate of other common-sense ideas under scientific scrutiny gives little confidence that the problems of conservation can be solved without a more fundamental approach than any that is here suggested.

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CALCULUS OF EXTERIOR FORMS

Forme Differenziali e loro Integrali

By Prof. Beniamino Segre. Vol. 1. Pp. 520. (Rome: Istituto Nazionale di Alta Matematica, 1951.) 3900 lire.

THE past twenty years have witnessed the final stages of what is perhaps the most remarkable synthesis in pure mathematics; during this period, a concept of multiplication, introduced by Grassmann and Hamilton more than a century ago, and now familiar to all in the algebra of vector products, has gradually penetrated analysis and geometry until what began as a convenient abridged notation and continued as a unifying force has ended in the creation of highly important branches of mathematics which include many classical theories as mere special cases.

In this field, known as the calculus of exterior forms, at least eight chief disciplines may be discerned, of which seven have recently converged to form the last. To begin with, there is the non-commutative algebra of Grassmann, with its interesting applications to geometry. Then there is the theory of systems of differential equations, including Pfaff's problem, transformation groups and integral invariants. Allied to this is Volterra's concept of functional, with the accompanying theory of integral equations and applications to Dirichlet's problem for any number of variables. The fourth group of subjects deals with generalizations to many complex

variables of the classical results due to Cauchy and Riemann. The next three groups are predominantly geometrical in inspiration. First, from Abel's theorem for an algebraic curve stem generalizations to integrals taken over surfaces and then algebraic manifolds of any dimension. Linked with this subject is Riemann's pioneer work on the topology of surfaces, later extended by Poincaré to general manifolds; and then, flowing from a quite different source, comes the stream of differential geometry, initiated by Gauss, generalized by Riemann, and reduced to a technique by the invention of the tensor calculus.

Within this generation it has been perceived, notably by E. Cartan, that the Grassmann algebra, applied to differential forms, could have a remarkable unifying effect upon the varied subjects described above. This discovery has led to the final synthesis: the theory of harmonic integrals, due to Hodge, deriving from de Rham, in which all the preceding material finds an essential place. (Only recently has it been noticed that these integrals are to be found in early work of Volterra.)

Until now it has been impossible for the non-specialist to acquire an adequate notion of this vast theory, some of which has never before appeared in book form, and most of which has not hitherto been expounded anywhere in the compact symbolism of the new calculus. The gap has at last been partly filled by the present lithographed work, which is based on lectures recently given in Rome. This, the first of two volumes, consists of four chapters, each sub-divided into numerous sections. The treatise opens with a short introduction which explains quite simply the nature and scope of the investigation. In the first chapter the Grassmann algebra is developed and applied to various problems of geometry. In Chapter 2 these concepts are brought to bear on the differential and integral calculus, the central point being the generalization of Stokes's theorem to any number of variables. The author then passes to the subject of differential equations, including infinitesimal transformations and integral invariants. Chapter 4 begins with a study of complex Euclidean space and the associated theory of complex variables, and thence proceeds to functionals and the general Dirichlet problem.

This brief summary, which mentions only the leading topics in the book, may well cause one to wonder how they could all be brought within the present compass. It is therefore the more striking that the treatment is not simply adequate but wholly comprehensive throughout; at no point does the author assume any special knowledge on the reader's part: on the contrary, he actually includes accounts of extraneous theories so as to render his exposition autonomous. Much of the latter is either original—for example, certain sections of Chapter 3, and the treatment of functionals—or, as in the account of functions of many complex variables, is derived from the work of Prof. Segre and his pupils.

At the end of each chapter there is a bibliographical note, composed of comments and references, which evinces a rare erudition. The work displays on every page the elegance and expository skill characteristic of the author: the wide territory is not merely surveyed, but surveyed from a single point of view. In fact, Prof. Segre's grasp of his subject is such that at any moment he can (and often does) call up any other of his themes so as to place it in relation to the matter in hand. This is a very important addition to the literature.

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