absolute number is given by A and must depend in some way on the anharmonicity of the inter-atomic forces. In principle, it is possible to derive A by perturbation theory from the anharmonicity as measured by the second pressure derivative in the equation of state; for example, a solution of a comparatively simple problem of this type has been given by Van Vleck[13]. Unfortunately, the difficulties of solving the relevant equations for the general problem of thermal conductivity appear to be very great; also it would not be easy to get the necessary experimental data for crystals as hard as sapphire. The density of helium, however, may be changed quite easily, and we hope that it may be possible to correlate its thermal conductivity with measurements on its compressibility.

To sum up, we may say that the thermal conductivities of pure dielectric crystals follow a common pattern. At sufficiently high temperatures they vary as $1/T$; then at lower temperatures the conductivities increase more rapidly until they are limited by size effect and fall off towards zero. The shape of the conductivity/temperature curve is to a fair approximation determined only by θ, the specific heat parameter. It is, of course, true that the values of the conductivity in the U-region also depend on the constant A, but as may be seen from Fig. 3 this lies within relatively narrow limits. Thus, while the magnitudes of the thermal conductivities of the different crystals vary greatly (the lowest value measured was 2×10^{-3} watt units for helium at 2° K. and the highest 60 watt units for sapphire at 50° K.), they can all be deduced, at least to an order of magnitude, once the specific heat parameter is known.

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THEORETICAL ASPECTS OF SUPERCONDUCTIVITY

Crystal Structure and Superconductivity

THE prediction that it is the interaction between electrons and lattice vibrations[1] which is responsible for the establishment of the superconductive state has been verified by the discovery of the isotope effect[2]. Theoretically, for a metal to be a superconductor a condition regarding the strength of this interaction has to be fulfilled. This condition requires (see ref. 1*a*, equations (3.19) and (6.8)) that a parameter which depends on the number ν of free electrons per atom must exceed a certain value. This parameter is proportional to $(4\nu)^{1/3}$ if $4\nu < 1$ and proportional to $1/(4\nu)^{1/3}$ if $4\nu > 1$. On the assumption that other quantities entering into it are independent of ν, an optimum for the attainment of superconductivity is reached if $\nu = 1/4$.

The number ν of free electrons of a metal depends on its Brillouin zone structure, which in turn is determined by the lattice structure. Since each zone contains sites for two electrons per unit volume, $\nu = 1/4$ means either nearly empty, or nearly full, zones. This is actually the case for most superconductors, as was remarked some time ago by Born and Cheng[3].

The result $\nu = 1/4$ was derived on the assumption of free electrons. It holds, however, whenever the energy of the electrons is proportional to the square of the wave-number. For some structures this may not be so, and thus a modification of the exact value of ν is required, although the above qualitative statement regarding the occupation of zones will probably hold in most cases. [June 9.

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Superconductivity and Effective Mass of Electrons

SOME time ago, Bardeen[1] suggested that a very small effective mass of electrons would lead to superconductivity. Later he found that the theory of the interaction between electrons and lattice vibrations[2] actually yields such a small effective mass[3]. I wish to show here, however, that small effective mass alone leads always to normal behaviour. Even though a state with non-penetrating magnetic field may be relatively stable[3], the normal state if it exists has always the lower Gibbs free energy, provided that no other than magnetic interaction between electrons is considered. Although the small effective mass is no doubt an important property, it is imperative in the presence of a magnetic field to consider the change of interaction between electrons and the vibrational field in order to obtain superconductivity, as has been shown recently[4].

To prove that without this the normal state exists (and hence is stable), let us review the results of the Landau–Peierls theory[5] of the diamagnetism of a gas of free electrons. Here the energy increase due to the presence of an external homogeneous magnetic field H_0 is calculated, based on a Hamiltonian:

$$\mathcal{H}_0 = \sum_k (\mathbf{p}_k - \frac{e}{c}\mathbf{A})^2/2m^*, \qquad (1)$$

where m^* is effective mass, $\mathbf{A}$ is the vector potential and the sum extends over all electrons. The result per unit volume is found to be

$$\frac{1}{2} \mid \chi_0 \mid (\mathrm{curl}\ \mathbf{A})^2, \qquad (2)$$

where the quantity $|\chi_0|$ is expressed in terms of various parameters. In particular, $|\chi_0| \to \infty$ if $m^* \to 0$. On the other hand, if χ is the magnetic susceptibility of a substance, its energy increases by $\frac{1}{2}|\chi|H_0^2$ if brought into the field, provided that $|\chi| \ll 1$. Hence, in this case χ_0 is equal to the magnetic susceptibility χ. Now since the permeability

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