All these dealt with the complex variable. Later came Cantor's theory of sets of points, leading to a great development of the theory of functions of a real variable. The name of Carathéodory is familiar to mathematicians as the author of a masterly treatise on the real variable. But the work under review is of quite a different nature; it can be described roughly as an introduction to a modern version of Riemann's geometrical ideas, freed from their original weaknesses, so that this approach can now be considered as firmly founded as the Weierstrassian. The main supports of this new version are Schwarz's theorems on conformal representation and Montel's concept of normal families of functions. These are used in conjunction with several modern developments, some due to the author himself, to give a clear and attractive exposition of fundamentals, though not including the general theory of Riemann's surfaces, of algebraic functions, or of uniformization. Although designed primarily as a text-book for students, some parts will be new and interesting even to the expert. The author died before it was published; but he had the satisfaction of seeing all the proofs, and his failing health would never have been suspected from the polished excellence of the work.

The division of the work into two volumes of unequal size, which is stated to be for the convenience of students, does not correspond to any real break in the continuity of the argument, and must have added to the cost of the work, which, at the present rate of exchange, is very high indeed in Great Britain. The first volume is divided into five parts, and the second volume into two more parts; each part is subdivided into three or four chapters. The first part begins with complex numbers, but takes an unexpected turn with the transition to spherical and non-Euclidean geometry. Emphasis is laid upon Ostrowski's *chordal difference* of two complex numbers, which is much used in the later pages, as it enables the point at infinity to be treated on the same footing as ordinary points. The second part gives a brief enunciation of such parts of the theory of sets of points as are needed in the later pages; for proofs the student is referred to the author's "Reelle Funktionen". The third part deals with analytic functions, including Cauchy's theorem, the maximum-modulus theorem, Schwarz's lemma, Poisson's integral, harmonic functions and meromorphic functions. The fourth part deals with the generation of analytic functions by limiting processes, using the somewhat unfamiliar ideas of *continuous convergence* and *limit-oscillation* to treat normal families of functions. These modern ideas precede the much older theorems of Taylor, Laurent, Mittag-Leffler, and a very brief treatment of the calculus of residues. The English-speaking reader may be puzzled why books in his own language contain so many examples, worked and unworked, on these topics, while Carathéodory's book contains none. The fifth part deals with some special functions, namely, the exponential and trigonometrical functions, the logarithm and general power, the gamma function, and Bernoulli's numbers.

Turning to the second volume, the sixth part begins with a discussion of functions the moduli of which in a certain domain do not exceed unity, and in particular those of which the moduli are equal to unity at every point on the circumference of a circle of unit radius. After the important theorems of Pick, Jensen, Julia, Lindelöf, Fatou and Riesz, the way is clear for a rigorous treatment of conformal representation. The seventh and last part opens with analytic functions of two or more variables and some existence theorems on differential equations. It then applies the conformal transformation of a triangle, the sides of which are circular arcs, to deal with the hypergeometric function and its differential equation. Next come the Schwarzian triangle-functions and the modular function. Finally, there is a discussion of essential singularities and the theorems of Landau, Schottky, Picard and Montel.

The work is beautifully printed, as well as beautifully written, and may be warmly commended to those who can afford to buy it.

H. T. H. Piaggio

THEORY OF PROBABILITY AND STATISTICS

Introduction to the Theory of Probability and Statistics

By Prof. Niels Arley and Prof. K. Rander Buch. (Applied Mathematics Series.) Pp. xi+236. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1950.) 32*s*. net.

THIS work is a translation (with a few alterations and additions) of the third Danish edition of a book which has already proved very successful in the country of its origin. For the most part, the translation has been carried out very well.

The two authors are well known for their achievements in different branches of the subject: Niels Arley has made important advances in the theory of stochastic processes and in its application to the problems of cosmic radiation, and K. R. Buch is best known for his work on the range of values assumed by a measure-function on an abstract space. When a physicist and a pure mathematician join forces, the outcome is sure to be interesting; in this case it is a highly individual book which should do good work in familiarizing students of the natural sciences with the ideas, notations and techniques of the modern theory of probability. Its excellence as an introductory text-book for physicists may be illustrated by mentioning that Poisson's distribution makes its first appearance in connexion with the Poisson process (stochastic processes in general appear on p. 48, and the ergodic theorem four pages later). No attempt is made to burden the book with proofs of the central-limit and allied major theorems; but their substance is sufficiently indicated, and the reader is given an accurate and clear picture of what can be expected from the mathematical theory. He will, if serious in his intentions, find this a useful introduction to a detailed study of the more comprehensive work by Feller.

As a text-book on statistics as opposed to probability theory, I found the book much less satisfactory, and it is perhaps a pity that a third collaborator was not found to prepare the 'Fisherian' chapters. For example, there is no mention of the analysis of variance, of the χ^2-test for 'goodness of fit' or of two-by-two contingency tables. In a subsequent edition there is much that could be done to improve these sections of the book; the first half is so good that such an improvement would be well worth while.

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