MATHEMATICS OF NON-LINEAR VIBRATIONS

Nonlinear Vibrations in Mechanical and Electrical Systems

By J. J. Stoker. (Pure and Applied Mathematics, Vol. 2.) Pp. xix+273. (New York and London: Interscience Publishers, Inc., 1950.) 40*s*.

Contributions to the Theory of Nonlinear Oscillations

By S. P. Diliberto, L. L. Rauch, F. H. Brownell, M. L. Cartwright, J. G. Wendel, C. E. Langenhop, A. B. Farnell, W. Wasow. Edited by S. Lefschetz. (Annals of Mathematics Studies, No. 20.) Pp. ix+350. (Princeton, N.J.: Princeton University Press; London: Oxford University Press, 1950.) 14*s*. 6*d*. net.

FOR more than two hundred years mathematicians have studied the properties of linear differential equations. In many problems, such as that of pendulum motion, where the equations are really non-linear, the linear approximations give a sufficiently good idea of what actually happens. However, engineers knew that many oscillatory mechanical devices had properties not explicable by the linear approximations; but until comparatively recently, these devices were small and inexpensive, and the easiest way to investigate them was by constructing and operating them. But now, in these days of aircraft, guided missiles and automatic chemical-process plants, it is an economic necessity to calculate in advance the possibly troublesome phenomena which may arise from the vibrations being non-linear. This work was at first undertaken chiefly by engineers and physicists. Now the pure mathematicians, especially in the United States and the U.S.S.R., have realized their opportunities in this field. They are pleasantly surprised to find how important the classical work of Poincaré, Hill and Mathieu turns out to be as a starting-point in attacking the new problems.

An excellent introduction to the subject is given by the first book, that by Prof. J. J. Stoker, whose lucid exposition gives continual references to particular mechanical or electrical systems from which arise the differential equations considered. The first chapter recalls the properties of linear vibrations. The second and third deal with free vibrations when the restoring force is non-linear, using a graphical discussion of the energy curve in the displacement–velocity plane, in which the time is taken as a parameter, and Poincaré's discussion of singular points is of fundamental importance. These methods, however, cannot be used when the time occurs explicitly in the differential equations, as for forced motion. The fourth chapter gives Duffing's apparently unnatural method for dealing with these problems, a method which succeeded when all straightforward approaches, even that of Rayleigh, had failed. This chapter includes also a number of useful analytical methods, in particular those of iteration and perturbation, and ends with a table showing some contrasts between linear and non-linear vibrations. The fifth chapter deals with non-linearity in the damping, which is responsible for 'self-sustained' oscillations, such as occur in radio valves, and in the 'flutter' of aircraft wings. The collapse of the Tacoma bridge is generally ascribed to these oscillations. A special analytical method for such problems was devised by Van der Pol, and this has been used extensively in the U.S.S.R. Other methods are due to Andronow and Witt, and to Miss Cartwright and Littlewood. It is rather surprising that the sixth chapter should deal with *linear* differential equations with periodic coefficients, but these are necessary for discussing the stability of non-linear oscillations. The classical results of Hill, Floquet and Mathieu thus gain a new importance. The appendixes give a rigorous mathematical justification of methods used in the main part of the book, which can be warmly recommended to engineers, physicists and mathematicians.

The second work under review is a collection of monographs edited by Prof. S. Lefschetz and will appeal chiefly to mathematicians engaged in research on the theoretical aspects of the subject, though many of the problems have a physical origin. S. P. Diliberto discusses some general questions connected with Poincare's results; L. L. Rauch deals with a differential equation of the third order arising from a problem concerning radio valves; and F. H. Brownell investigates the oscillatory solutions of a difference-differential equation, such as arise in control problems. The longest paper is by Miss M. L. Cartwright, and deals with forced oscillations. Similar problems are treated in a different way in papers by J. G. Wendel, by C. E. Langenhop and A. B. Farnell, and by W. Wasow. This volume, like other Princeton University publications, deserves careful study by research workers.

H. T. H. PIAGGIO

MECHANICAL PROPERTIES OF METALS AT HIGH AND LOW TEMPERATURES

The Properties of Metallic Materials at Low Temperatures

By P. Litherland Teed. (Monographs on Metallic Materials, published under the authority of the Royal Aeronautical Society, Vol. 1.) Pp. viii+222. (London: Chapman and Hall, Ltd., 1950.) 21*s*. net.

Properties of Metals at Elevated Temperatures

By George V. Smith. (Metallurgy and Metallurgical Engineering Series.) Pp. x+401. (London: McGraw-Hill Publishing Co., Ltd., 1950.) 59*s*. 6*d*.

THE wide temperature range, some 1,100 deg. C., over which the engineer is concerned with the mechanical properties of metals and alloys is covered by these two volumes.

"The Properties of Metallic Materials at Low Temperatures" is the first of a series of monographs to be published under the authority of the Royal Aeronautical Society, and contains a survey of existing knowledge regarding the non-ferrous metals and their alloys, and the steels. The bringing together of this information is justified not only in connexion with the construction of planes for high-altitude flight, but also by the increasing use of low temperatures by the chemical engineer. Chapters are devoted to a consideration of the major mechanical properties of aluminium, iron, magnesium, copper, nickel, zinc, tin and lead and their alloys, about one-half being devoted to the plain carbon and alloy steels. The author points out that the variations of properties at very low temperatures are as yet unpredictable, and makes the further most pertinent point that the fruits of long and careful work are too often wasted as a result of inadequate description of the composition, and the mechanical- and heat-treatment of the samples investigated.