

LETTERS TO THE EDITORS

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Nuclear Shell Structure and Nuclear Density

THE existence of the shell structure of nuclei has been derived from a considerable variety of empirical facts (stability and abundance of nuclei, spin, magnetic moment, quadrupole moment, isomerism and β -decay)¹. The closed shells of neutrons as well as of protons correspond to the numbers 2, 8, 20, 28, 50, 82, 126. Several theories² have been put forward, based on the model of a single nucleon moving in an average field of the rest. As it is not easy to conceive how a single particle embedded in matter of nuclear density could move in its own orbit without being disturbed by the others, we have searched for a more general explanation*. This is obtained with the help of a well-known relation first derived by Fermi, connecting the density $\rho(r)$ of a degenerate system of spin-half particles to the numbers of particles with angular momentum quantum number l . If one assumes that the formation of a closed shell is associated with the filling up of states of definite angular momentum, it follows that the closed shells are characterized by

$$[r^3 \rho(r)]_{\text{max}} = (2l + 1)^3 / 24\pi^2.$$

Then calculations performed by the junior author show that the number of particles $N(l)$ in the l -th shell is

$$N(l) = f(2l + 3)^3,$$

where f is a quantity depending on the form of the density function $\rho(r)$. One sees that the difference of the cubic roots of successive shells $\Delta(N^{1/3})$ is a constant; namely, $2f^{1/3}$. Now this is at once empirically confirmed by the shell numbers in heavy nuclei, as shown in the following table:

$N(l)$	=	28	50	82	126
$\{N(l)\}^{1/3}$	=	3.04	3.69	4.35	5.01
$\Delta(N^{1/3})$	=		0.65	0.66	0.66

One can now calculate theoretically the shell numbers for different density distributions. The free Fermi-gas distribution gives too small a value of $\Delta(N^{1/3})$, equal to 0.52 instead of 0.66. However, a reasonable density distribution $\rho(r)$, constant in the interior and falling off like a Gaussian function near the border, namely,

$$\rho(r) = \begin{cases} \rho_0 & \text{for } r \leq R_0 \\ \rho_0 \exp \left\{ -\left(\frac{r-R_0}{a} \right)^2 \right\} & \text{for } r > R_0 \end{cases} \quad (1)$$

gives excellent result for $l \gg 3$. The constants ρ_0 , a and R_0 , determined uniquely by the empirical relation $\Delta(N^{1/3}) = 0.66$, are

$$\left. \begin{aligned} \rho_0 &= 1.04 \frac{4\pi}{3} r_0^3, \\ a &= 0.327 r_0 A^{1/3}, \\ R_0 &= 0.673 r_0 A^{1/3}. \end{aligned} \right\} \quad (2)$$

where r_0 is of the order of 1.5×10^{-13} cm.

The shell numbers $N(l)$ as calculated from (1) and (2) are tabulated in the following table:

l	=	3	4	5	6
$N(l)$	=	27.1	49.5	81.5	125.2
Next integer	=	28	50	82	126

For light nuclei, the central part of constant density no longer exists. When a purely Gaussian function is adopted for the nuclear density,

$$\rho(r) = \rho_0 \exp \left\{ -\left(\frac{r}{a} \right)^2 \right\}, \text{ with } \rho_0 = 0.752 \frac{4\pi}{3} r_1^3, \text{ and } a = r_1 A^{1/3},$$

where r_1 is also of the order of 1.5×10^{-13} cm.

We again obtain correctly the first three shell numbers:

l	=	0	1	2
$N(l)$	=	1.55	7.16	19.66
Next integer	=	2	8	20

There must exist, of course, an intermediate region, presumably between 20 and 28, in which the transition takes place. Theoretical calculation of the nuclear density according to Dirac and Jensen's refinement of the Thomas-Fermi model of nuclei is now in preparation.

Thus the shell structure, owing to its correlation with the nuclear density, throws light on the possible form of the latter. In particular, it turns out that the thickness of the surface layer, instead of being constant and of the order of the nuclear-force range, has to be proportional to $A^{1/3}$. We further infer that the existence of a central part of almost constant density is due to the effect of the Coulomb repulsion, without which the nuclear density would be nearly Gaussian for all nuclei.

Detailed calculation will be published elsewhere by the junior author.

*Note added in proof. After having finished this paper, we found that the problem has been tackled in a similar way by Iwanenko and Polyjew³, but with less success; the reason being that only ordinary forces of the Yukawa type are taken into account while actually exchange forces are predominant.

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¹ Elsassner, W., *J. de Phys. et le Rad.*, **5**, 625 (1934). Wigner, E., *Phys. Rev.*, **51**, 947 (1937). Mayer, Maria G., *Phys. Rev.*, **74**, 235 (1948).

² Feenberg, E., and Hammack, K. C., *Phys. Rev.*, **75**, 1877 (1949). Nordheim, L. W., *Phys. Rev.*, **75**, 1894 (1949). Mayer, Maria G., *Phys. Rev.*, **75**, 1969 (1949). Hazen, Otto, Hans, J., Jensen, D., and Suess, Hans E., *Phys. Rev.*, **75**, 1766 (1949).

³ Iwanenko, D., and Polyjew, W., *Dokl. Akad. Nauk, S.S.S.R.*, **70** (No. 4), 605 (1950).

Microscopy by Reconstructed Wave-fronts

THE principle of Dr. D. Gabor's interesting method of reconstructing images of objects from photographs of the interference patterns produced when the object is illuminated with a coherent monochromatic wave-train is fully explained in his paper¹. The treatment in this note is essentially the same, but I have ventured to present it in a very simplified form because I have found in discussions that difficulty is sometimes experienced in forming a physical picture of the reconstruction of the image, when the photographic plate can only record intensities of light and not phases.

The object O is placed close to a point source of light S . The wavelets scattered by the object interfere with the main waves from S over a surface such