

normally slow. They have a shiny chitinous body-wall, a wing-like extension (the pteromorpha) at the antero-lateral corner of the abdomen on each side, a pair of peculiarly shaped sense-organs (the pseudo-stigmatic organs), and no eyes.

By and large denizens of the soil, the oribatid mites occur in decaying vegetation, among fungal mycelia, moss or algae, and form a conspicuous group amid the soil biota. Some are confined to the soil and migrate on blades of grass during the night; others conceal themselves in the bark of trees or crawl on twigs and leaves. Depending upon their habitat, they are classified as terricolous, dendricolous, bryobious and humicolous.

In the group Acarina, which comprises parasites (Ixodoidea and Sarcoptidæ), predators (Trombidiidæ), and suckers of plant sap (Tetranychidæ), the Oribatoidea are outstanding in being herbivorous or saprophagous. They feed on moulds (fungivorous), algae, decaying leaves or animal matter, and are distinctly inoffensive. Consequently, although systematists like Banks, Berlese, Nicolet, Michael, Oudemans, Jacot, Grandjean and Ewing have revealed the abundant and ubiquitous occurrence of these organisms, very few investigators attributed any importance to them. Michael², in his monograph, has remarked that none of the species is "injurious to man or his works". Jacot³ regarded them as "negative, humble and devoid of weapons of offense"; afterwards⁴, he commented on the occurrence of *Damæosoma alces*, *Protoschelobates pembertonii* and *Xylobates pembertonii* in holes in cane roots, of *Lohmannia insignis* on bean seedlings and of *L. insignis dissimilis* on tulip bulbs as having no significance, because no evidence could be adduced that the holes were caused by the mites. He concluded that "the case against oribatides as non-injurious is becoming weightier".

Stunkard's⁵ experiments on the development of *Moniezia expansa* disclosed the identity of the intermediate host of this tapeworm. He announced that *Zetes (Galumna) emarginatus* harboured the cysticercoids of the worm to the infective stage, and this discovery heralded a series of publications on the life-cycles of other Anoplocephalid cestodes. The work of Krull⁶ (U.S.A.) on *Cittotænia* and *Moniezia*, of Stunkard⁷ (U.S.A.) on *Bertiella studeri*, of Pullar⁸ (Australia) on *Moniezia*, of Bashkirova⁹ (U.S.S.R.) on *Anoplocephala* spp., and *Paranoplocephala mamillana*, of Potemkina¹⁰ (U.S.S.R.) on *Thysaniezia ovilla*, and of the present writer¹¹ (India) on *Moniezia expansa* and *M. benedeni*, has involved oribatid mites of various genera and species as the undoubted vectors of these worms. Moreover, the same species of mite is observed to be capable of conveying more than one species of the cestode in a locality.

The Anoplocephalid tapeworms, characteristically devoid of rostellar hooks, are among the worst enemies of livestock, and to the veterinary parasitologist the oribatid mite has therefore suddenly become of major importance.

¹ *Nature*, 156, 427 (1945).

² "British Oribatoida", 1 (1884).

³ *Amer. Nat.*, 59, 272 (1925).

⁴ *Bern. P. Bishop Mus. Bull.*, 121, 93 (1934).

⁵ *Parasit.*, 30, 491 (1938).

⁶ *Proc. Helm. Soc. Wash.*, 6, (1), 10 (1939).

⁷ *Amer. J. Trop. Med.*, 20 (2), 305 (1940).

⁸ *Austral. Vet. J.*, 15, (2), 71 (1939).

⁹ *C.R. Acad. Sci. URSS.*, (N.S.) 30, (6), 576 (1941) (in English). *Vestn. Sel.-khoz. Nauk., Veterinariya*, No. 2, 57 (1941, in Russian, with French summary).

¹⁰ *C.R. Acad. Sci. URSS.*, (N.S.), 30, (5), 474 (1941); 43, (1), 43 (1944).

¹¹ Unpublished work.

THE MESON FIELD AND THE EQUATION OF MOTION OF A SPINNING PARTICLE

By R. C. MAJUMDAR and DR. S. GUPTA
Tata Institute of Fundamental Research, Bombay

THE outstanding difficulty in obtaining the equation of motion of a spinning particle in a meson field which will take account of the reaction of the emitted meson field and is free from singularities is well known. The equation of motion was first obtained by Heisenberg¹ for a dipole of finite extension and is therefore not relativistically invariant. The relativistically invariant equations of motion for a point dipole which have been developed by Bhabha and Corben² and by Bhabha³ are extremely complicated and involve a number of arbitrary constants which are not determined uniquely. These equations in their final form are free from singularities, and have been proved to involve only the contribution of the so-called radiation field, defined as half the retarded minus the advanced field, which is finite on the world line. However, the equations are established by proving that the singular terms are perfect differentials and can therefore be subtracted away.

In 1939, Riesz⁴ developed an elegant method for solving the differential equations of hyperbolic type in which the divergent difficulties are avoided by a process of analytical continuation. The method has been recently applied by Fremberg⁵ to obtain the classical equation of motion of a charged particle in an electromagnetic as well as in a meson field. Following Riesz, we shall define the potential and the field of a dipole for a parameter $\alpha > 2$ and show that the analytical continuation to $\alpha = 2$ gives the classical values corresponding to those of Lienard-Wiechert outside the world line, and a finite value on the world line itself giving the reaction of the field. The equations of motion of a point dipole can then be obtained in a straightforward way from the usual equation of a dipole as given by Frenkel⁶. The equations are free from singularities and do not contain any arbitrary constant except the spin angular momentum of the dipole.

We define the Riesz potential for the dipole at a point $P(x)$ outside the world line in the form,

$$\varphi_{\nu}^{\alpha}(P) = \frac{g_{\nu} \chi^{\alpha} - \alpha}{2^{\frac{1}{2}}(\alpha - 2) \Gamma(\alpha/2)} \frac{\partial}{\partial x_{\mu}} \int_{-\infty}^{\tau_0} S_{\mu\nu}(\tau) (\chi s)^{\frac{1}{2}(\alpha - 4)} J_{\frac{1}{2}(\alpha - 4)}(\chi s) d\tau, \quad (1)$$

where τ is the proper time of the particle, τ_0 the retarded point of P , $J_n(x)$ is the Bessel function of order n and $g_{\nu} S_{\mu\nu}$ is the antisymmetrical tensor describing the dipole moment. The four-dimensional distance s is given by

$$s^2 = s_{\mu} s^{\mu}, \quad s_{\mu} = x_{\mu} - z_{\mu}. \quad (2)$$

The integral converges for $2 < \alpha < 5$. The potential (1) satisfies the wave equation of a dipole in a meson field given by

$$\frac{\partial}{\partial x_{\rho}} \frac{\partial}{\partial x^{\rho}} \varphi_{\mu} + \chi^2 \varphi_{\mu} = 4\pi \frac{\partial}{\partial x_{\nu}} S_{\nu\mu}, \quad (3)$$

when we continue analytically to $\alpha = 2$.

Now it can be shown after some calculations that the analytical continuation to $\alpha = 2$ of the Riesz potential (1) and the corresponding field-strength for

the dipole lead to the well-known Lienard - Wiechert results, as already obtained by Bhabha and Corben :

$$\varphi_{\nu}^2(P) = \varphi_{\nu}^{\text{ret}} = g_2 \left[\frac{1}{\kappa} \frac{d}{d\tau} \left(\frac{s^{\mu} S_{\mu\nu}}{\kappa} \right) \right]_0 - \frac{g_2 \chi^2}{2} \left[\frac{s^{\mu} S_{\mu\nu}}{\kappa} \right]_0 + g_2 \chi^2 \int_{-\infty}^{\tau_0} \frac{s^{\mu} S_{\mu\nu}}{s^2} J_2(\chi s) d\tau, \quad (4)$$

$$F_{\mu\nu}^2(P) = F_{\mu\nu}^{\text{ret}} = g_2 \left[\frac{1}{\kappa} \frac{d}{d\tau} \left(\frac{S_{\mu\nu}}{\kappa} \right) \right]_0 + g_2 \left[\frac{1}{\kappa} \frac{d}{d\tau} \left(\frac{1}{\kappa} \frac{d}{d\tau} \left(\frac{T_{\mu\nu}}{\kappa} \right) \right) \right]_0 - \frac{g_2 \chi^2}{2} \left[\frac{S_{\mu\nu}}{\kappa} \right]_0 - \frac{g_2 \chi^2}{2} \left[\frac{1}{\kappa} \frac{dT_{\mu\nu}}{d\tau} \right]_0 + \frac{g_2 \chi^4}{8} \left[\frac{T_{\mu\nu}}{\kappa} \right]_0 + g_2 \chi^2 \int_{-\infty}^{\tau_0} \frac{S_{\mu\nu} J_2(\chi s)}{s^2} d\tau - g_2 \chi^2 \int_{-\infty}^{\tau_0} \frac{T_{\mu\nu} J_3(\chi s)}{s^3} d\tau - \text{the same terms with } \mu \text{ and } \nu \text{ interchanged, } \dots \dots (5)$$

where $\kappa = s_{\mu} v^{\mu}$, $T_{\mu\nu} = s_{\mu} s^{\sigma} S_{\sigma\nu}$. The suffix 0 implies that the value of the function is to be taken at the retarded point of P.

In order to calculate the potentials and the fields at a point P on the world line, we divide the integrals into two parts, one in the interval $-\infty$ to $\tau_0 - \epsilon$, and the other from $\tau_0 - \epsilon$ to τ_0 . It is easily seen that the first integral vanishes for $\alpha = 2$. We then expand the quantities in the second integral in power series of τ , measured from τ_0 taken as zero, and obtain by analytical continuation to $\alpha = 2$ the following expressions :

$$\varphi_{\nu}^{\alpha=2} \text{ (on the world line)} = -g_2 \left[\frac{1}{3} v^{\mu} S_{\mu\nu} - \frac{1}{3} v^{\mu} S_{\mu\nu} (\dot{v}\dot{v}) + \dot{v}^{\mu} \dot{S}_{\mu\nu} + v^{\mu} \dot{S}_{\mu\nu} \right] - \frac{1}{2} g_2 \chi^2 v^{\mu} S_{\mu\nu} + g_2 \chi^2 \int_{-\infty}^{\tau_0} s^{\mu} S_{\mu\nu} \frac{J_2(\chi s)}{s^2} d\tau, \quad (6)$$

$$F_{\mu\nu}^{\alpha=2} \text{ (on the world line)} = g_2 \left[\frac{2}{3} S_{\mu\nu} (\dot{v}\dot{v}) + \frac{2}{3} \dot{S}_{\mu\nu} \dot{v}^2 + \frac{2}{3} \ddot{S}_{\mu\nu} - \left\{ \frac{4}{3} v_{\mu} S_{\nu}''' + 2v_{\mu} S_{\nu}' \dot{v}^2 + \frac{1}{3} \ddot{v}_{\mu} S_{\nu} + \frac{2}{3} \dot{v}_{\mu} \dot{v}^{\sigma} S_{\sigma\nu} + \frac{4}{3} \dot{v}_{\mu} S_{\nu}' + \frac{4}{3} v_{\mu} \dot{v}^{\sigma} \dot{S}_{\sigma\nu} + \frac{2}{3} \dot{v}_{\mu} \dot{v}^{\sigma} S_{\sigma\nu} + \frac{1}{3} v_{\mu} \ddot{v}^{\sigma} S_{\sigma\nu} + 2v_{\mu} \dot{v}^{\sigma} \ddot{S}_{\sigma\nu} + 2\dot{v}_{\mu} S_{\nu}'' + 2\dot{v}_{\mu} \dot{v}^{\sigma} \dot{S}_{\sigma\nu} + 2v_{\mu} \dot{v}^{\sigma} S_{\sigma\nu} (\dot{v}\dot{v}) + v_{\mu} v^{\sigma} S_{\sigma\nu} \dot{v}^2 + \dot{v}_{\mu} S_{\nu} \dot{v}^2 \right\} - \chi^2 \left\{ \dot{S}_{\mu\nu} - [v_{\mu} S_{\nu}'] - \frac{1}{2} [v_{\mu} \dot{v}^{\sigma} S_{\sigma\nu}] - \frac{1}{2} [\dot{v}_{\mu} S_{\nu}] \right\} \right] + 2g_2 \chi^2 \int_{-\infty}^{\tau_0} S_{\mu\nu} \frac{J_2(\chi s)}{s^2} d\tau - g_2 \chi^3 \int_{-\infty}^{\tau_0} [s_{\mu} s^{\sigma} S_{\sigma\nu}] - \frac{J_3(\chi s)}{s^3} d\tau, \dots \dots (7)$$

where $S_{\nu} = v^{\sigma} S_{\sigma\nu}$, $S_{\nu}' = v^{\sigma} \dot{S}_{\sigma\nu}$, $S_{\nu}'' = v^{\sigma} \ddot{S}_{\sigma\nu}$, $S_{\nu}''' = v^{\sigma} \ddot{S}_{\sigma\nu}'$, . . . (8)

and the minus sign as subscript denotes that the terms inside the brackets are to be subtracted with interchange of μ and ν .

It is to be noticed that the field given by (7) is, in fact, the retarded field, and agrees with the

so-called radiation field of Dirac as obtained by Bhabha and Corben, provided we put $k_1 = -1/3$, $k_2 = -7/15$, $k_3 = 1/3$, $k_4 = 4/3$, $k_5 = 0$ in (140) (Bhabha and Corben) and $k = 2$ in (110) (Bhabha). There is, however, disagreement in terms with constant k_5 , which has also been noticed by Harish-Chandra⁷, whose calculations of the radiation field on the world line by retaining the finite and the unambiguous terms of the fields agrees with our result (7).

The equation of motion is now obtained from Frenkel's equation

$$I \dot{S}_{\mu\nu} = g_2 [S.F^{\text{ext}}]_{\mu\nu} + g_2 [S.F^{\text{react}}]_{\mu\nu}, \quad (9)$$

where I is the spin angular momentum. The field $F_{\mu\nu}$ in the second term of the right-hand side is given by (7) above. The equation of motion of a point dipole in a scalar meson field is also similarly obtained.

The detailed paper will be published in due course in the *Physical Review*.

¹ Heisenberg, W., *Z. Phys.*, **113**, 61 (1939).
² Bhabha, H. J., and Corben, H. C., *Proc. Roy. Soc., A*, **178**, 273 (1941).
³ Bhabha, H. J., *Proc. Roy. Soc., A*, **178**, 314 (1941).
⁴ Riesz, M., Conférence de la Réunion internat. des math. tenue a Paris en Juillet 1937 (Paris, 1939). Prof. Riesz's papers are unfortunately not available in India.
⁵ Fremberg, N. E., *Medd. Lunds Univ. Mat. Sem.*, **7** (1940) *Proc. Roy. Soc., A*, **188**, 18 (1946).
⁶ Frenkel, J., *Z. Phys.*, **37**, 243 (1926).
⁷ Harish-Chandra, *Proc. Roy. Soc., A*, **185**, 269 (1946).

FORTHCOMING EVENTS

(Meetings marked with an asterisk * are open to the public)

Monday, March 15

ROYAL PHYSICAL SOCIETY OF EDINBURGH (at the Royal Scottish Geographical Society, Synod Hall, Castle Terrace, Edinburgh), at 5.15 p.m.—Prof. T. N. George: "Fossils and Evolutionary Theory".

MANCHESTER LITERARY AND PHILOSOPHICAL SOCIETY (in the Reynolds Hall, College of Technology, Manchester), at 5.30 p.m.—Sir Charles Goodeve, F.R.S.: "Operational Research".*

UNIVERSITY COLLEGE (in the Anatomy Theatre, Gower Street, London, W.C.1), at 5.30 p.m.—Dr. George Sarton: "Science and Tradition". (Further Lectures on March 16 and 19)*

ROYAL INSTITUTE OF CHEMISTRY, LONDON AND SOUTH-EASTERN COUNTIES SECTION (joint meeting with the DARTFORD BRANCH OF THE PHARMACEUTICAL SOCIETY, at the County Technical College, Essex Road, Dartford), at 7.15 p.m.—Dr. A. Albert: "Physico-Chemical Concepts in Interpreting Drug Action".

Tuesday, March 16

BRITISH PSYCHOLOGICAL SOCIETY, INDUSTRIAL SECTION (at the London School of Hygiene and Tropical Medicine, Keppel Street, London, W.C.1), at 1 p.m.—Annual General Meeting: Mr. Dermot Straker: "The Importance of Information in Vocational Selection".

CHADWICK PUBLIC LECTURE (in the Sir Edward Moyerstein Lecture Theatre, Westminster Hospital Medical School, 17 Horseferry Road, London, S.W.1), at 2.30 p.m.—Dr. J. R. Nicholls: "Adulteration of Food and its Detection".*

ROYAL SOCIETY OF ARTS, DOMINIONS AND COLONIES SECTION (at John Adam Street, Adelphi, London, W.C.2), at 2.30 p.m.—Dr. E. Marsden, F.R.S.: "Co-ordination of Research in the Pacific".

LINNEAN SOCIETY OF LONDON (joint meeting with the SYSTEMATICS ASSOCIATION, at Burlington House, Piccadilly, London, W.1), at 5 p.m.—Discussion on "The Organization of Taxonomic Research".

ROYAL ANTHROPOLOGICAL INSTITUTE (at 21 Bedford Square, London, W.C.1), at 5 p.m.—Miss Laura E. Start: "Indian Textiles from Guatemala and Southern Mexico".

ROYAL STATISTICAL SOCIETY (at the London School of Hygiene and Tropical Medicine, Keppel Street, London, W.C.1), at 5.15 p.m.—Mr. F. J. Anscombe: "The Validity of Comparative Experiments".

EUGENICS SOCIETY (at the Royal Society, Burlington House, Piccadilly, London, W.1), at 5.30 p.m.—Dr. W. Mayer Gross: "Mental Health Survey in a Rural Population".*

CONWAY DISCUSSION CIRCLE (at Conway Hall, Red Lion Square, London, W.C.1), at 7 p.m.—Dr. W. E. Swinton: "The Rise and Fall of the Dinosaurs".*

INSTITUTION OF THE RUBBER INDUSTRY (at Caxton Hall, Caxton Street, London, S.W.1).—Dr. D. A. Harper: "The Technology of some New Condensation Rubbers".

Tuesday, March 16—Thursday, March 18

INSTITUTE OF METALS (at the Institution of Civil Engineers, Great George Street, London, S.W.1).—Annual General Meeting.