LETTERS TO THE EDITORS

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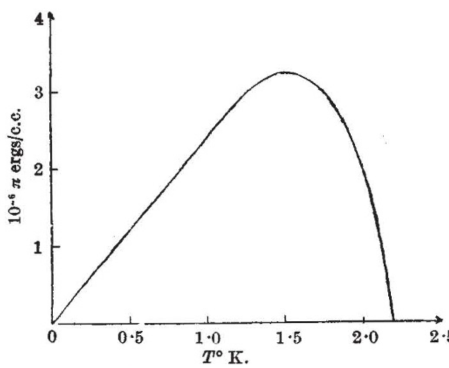
Liquid Helium II

THE classical theory of liquids[1,2,3], of which a general explanation was given by Prof. Max Born and me in a previous article[4], and the quantization of the theory[5,6] which followed, led to a conception of the nature of viscosity and thermal conduction in liquids which we hoped would contribute towards the understanding not only of normal liquids, but also of 'quantum liquids', of which helium II and the electrons in superconducting metals are known examples.

The detailed application of these results to helium II has now been completed, and it has been found possible, without further assumption, to explain all the well-known properties of this liquid. The essential feature of a quantum liquid, as we observed previously[6], is the distinction between the intensive properties defined thermodynamically and from the point of view of kinetic theory. Of special importance is the distinction between the 'thermodynamic' pressure p, defined in terms of the work done pdV during the displacement of a surface in the liquid, and the 'kinetic' pressure p_1, the gradient of which determines the mean motion of the molecules. It can be shown that the difference π between these two quantities first becomes numerically significant just below the λ-point; it is represented as a function of temperature in the accompanying figure.



π AS A FUNCTION OF TEMPERATURE AT THE SATURATION VAPOUR PRESSURE

The equation of motion of the quantum liquid is

$$\rho \frac{du}{dt} + \frac{\partial p}{\partial x} = - \frac{\partial \pi}{\partial x}, \qquad (1)$$

where ρ is the density and u the macroscopic velocity. It follows rigorously that a reformulation of the first law of thermodynamics is required below the λ-point, of such a kind that it reduces to the usual law

$$dQ = dU + pdV \qquad (2)$$

for the ideal quasi-static process, but has quite a different form

$$dQ = d(U - \pi V) + p_1 dV \qquad (3)$$

for steady motion, such as is observed in the transfer effect (studied in detail by Daunt and Mendelssohn[7]), and yet another form

$$dQ = d(U + \pi V_0 - \pi V) + p_1 dV \qquad (4)$$

for wave motion, V_0 being the rest volume of the liquid. The anomalous specific heat and density of helium II is thus explained by the presence of thermal waves. Similar waves have been observed in ordinary liquids[8], but are strongly damped and have only a small group velocity.

A thermal wave transfers heat energy and the liquid bulk in opposite directions in a ratio depending only on the temperature. Kapitza's experiments[9] on the thermomechanical effect in a narrow slit have a quantitative explanation not, as Landau[10] suggests, in the motion of a superfluid at absolute zero, but in the almost isothermal passage of a thermal wave through the slit. Peshkov's observations[11] of temperature waves in helium II also have a quantitative explanation in terms of the thermal waves.

Earlier theories due to F. London[12,13] and Tisza[14,15] suggest that superfluidity is connected with the condensation which occurs in a Bose gas not far from the λ-point. A serious objection to these theories is the occurrence of entirely analogous phenomena in superconductors, where the electrons obey Fermi statistics. The quantum theory of liquids is, however, equally applicable to superconductivity, and an interesting feature of the theory outlined here is that it can explain also phenomena in superconductors: one requires only to replace the mechanical pressure by the electromagnetic stress tensor and to represent the almost stationary metallic ions by a conservative field of force.

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[1] Born, M., and Green, H. S., *Proc. Roy. Soc.*, A, **188**, 10 (1946).
[2] Green, H. S., *Proc. Roy. Soc.*, A, **189**, 103 (1947).
[3] Born, M., and Green, H. S., *Proc. Roy. Soc.*, A, **190**, 455 (1947).
[4] Born, M., and Green, H. S., *Nature*, **159**, 251 (1947).
[5] Born, M., and Green, H. S., *Proc. Roy. Soc.*, A, **191**, 168 (1947).
[6] Born, M., and Green, H. S., *Nature*, **159**, 738 (1947).
[7] Daunt, J. G., and Mendelssohn, K., *Proc. Roy. Soc.*, A, **170**, 423 (1939).
[8] Lucas, R., *J. de Phys.*, (7), **9**, 410 (1937).
[9] Kapitza, P. L., *J. Phys. U.S.S.R.*, **5**, 59 (1941).
[10] Landau, L., *J. Phys. U.S.S.R.*, **5**, 71 (1941).
[11] Peshkov, V., *J. Phys. U.S.S.R.*, **10**, 389 (1946).
[12] London, F., *Phys. Rev.*, **54**, 947 (1938).
[13] London, F., *J. Phys. Chem.*, **43**, 49 (1939).
[14] Tisza, L., *C.R. Acad. Sci., Paris*, **207**, 1035, 1186 (1938).
[15] Tisza, L., *J. de Phys.*, (8), **1**, 165, 350.

Theory of Skating

IT is perhaps less common than it was to find the phenomenon of regelation put forward as a complete explanation of skating. Very probably anyone with a scientific education will suspect that plastic flow in the solid ice plays an important part, and the following observations will, if they are accepted, serve to strengthen that opinion.

It is clear at once that regelation as ordinarily understood in this case must require a rapid flow of heat through the skate blade from back to front, and yet the temperature gradient must be an infinitesimal one. Obviously the effect of friction must also be included, and since the publication of Bowden and Hughes' paper[1] in 1939 it has become possible to calculate what this may be. What no one seems to know with any accuracy is the depth of penetration of the skate blade and the amount of ice actually displaced. This is not easy to determine experimentally, and for present purposes the results have been