charge do not enter within this framework. They are implicit in the values assumed by V, R and S for a given value of Q the discharge. These seven variables demand the following four dimensionless arguments :

$$V^2/gRS\ ;\ V^2/gR\ ;\ RV/\mu\ ;\ P/R,$$

in which the slope S has been inserted by substitution.

Of these four arguments any one may be expressed as an empirical function of any two of the remaining three. Two equations thus afford a complete solution, and they may be written :

$$P/R = \tfrac{1}{2}\,(V^2/gR)^{1/3}\,(RV/\mu)^{1/3} \qquad (A)$$
$$V^2/gRS = \tfrac{1}{4}\,(V^2/gR)^{-2/3}\,(RV/\mu)^{1/3}. \qquad (B)$$

These two equations reduce to

$$P = 2\cdot78Q^{1/2},$$

and

$$V = 15\cdot8R^{2/3}\,S^{1/3},$$

when the temperature of the water is 30° C.

The coefficients $\frac{1}{2}$ and $\frac{1}{4}$ respectively in the two basic equations appear to be pure numbers under ideal conditions.

On elimination of the Reynolds number from the basic equations, my equation published in *Nature* of August 3, p. 166, immediately results, namely:

$$V^2/gRS = \tfrac{1}{2}\,(gR/V^2)\ .\ P/R$$
$$= \tfrac{1}{2}\ gP/V^2. \qquad (C)$$

The manner in which the three arguments enter should be noted. The new Froude number V^2/gP is none other than the usual Froude number V^2/gR divided by the shape number P/R. Similarly the 'White' number $ag^{2/5}/Q^{4/5}$ raised to the fifth power is none other than $P/R\ .\ (gR/V^2)^2$.

It would appear desirable, when making an attack on any dimensional problem, to express the new dimensionless numbers involved, if possible, in terms of more familiar dimensionless arguments. When this is done, the precise nature of the assumptions made is more readily appreciated.

The difficulties experienced in the past in the dimensional treatment of alluvial channels in loose material have arisen from the fact that the full implications of an ideal self-generating channel have not been understood. Old methods have been employed to solve a new problem. The reappearance of the old familiar dimensionless arguments, or numbers, is, however, reassuring.

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Submarine Tubes for Levelling

THE ocean surface deviates from the geoid, that is, from a level surface, by an amount probably exceeding one metre even if short-period disturbances such as waves and tides are eliminated. It therefore cannot serve as a datum for precise levelling. Coasts separated by water can, however, be joined by hydrostatic levelling, recently described by Nørlund. In 1938 he 'bridged' the Store Belt[1] in Denmark with a water-filled submarine tube of length 18 km. and inside diameter 1 cm. ; the greatest depth was 60 m. Observations of atmospheric pressure at each end and of temperature distribution along the tube served to determine the difference in level between the water surfaces inside each end. In 1939 he joined the spirit-level networks of Denmark and Sweden by means of a 4·2 km. submarine tube beneath Oresund[2].

Nørlund's new method can furnish information needed in physical oceanography. The relations between current and slope of the ocean surface, except the part due to difference in atmospheric pressure, are the same as the more widely known relations between wind and slope of an isobaric surface in the atmosphere. Previous knowledge about the level or geopotential of the ocean surface has depended entirely upon the spirit-level method and the oceanographic method. The latter method gives the difference in geopotential between point 1 and point 2 in the form

$$\varphi_2 - \varphi_1 = -\int_{p_1}^{p_2}\alpha\ dp\ ;$$

the integral of specific volume α with respect to pressure p from point 1 to point 2 must be carried out along an instantaneous path consisting only of (a) vertical lines, and (b) lines along which the horizontal component of the pressure gradient vanishes. The distribution of specific volume is obtained by ordinary oceanographic methods. The path is so chosen that non-vertical parts are at great depths, where condition (b) can be expected to be approximately fulfilled.

Hydrostatic levelling is similar to oceanographic levelling, but has the advantage that the water in the tube comes to rest or oscillates about a position of equilibrium, so that condition (b) is necessarily fulfilled. The difference in geopotential between the two ends can, therefore, be computed from the equation above. Since the tube is filled with a liquid of known pressure-volume-temperature relationship, the necessary data are obtained by measuring the pressures p_1 and p_2 inside the ends, and the distributions of temperature and of approximate depth along the tube.

Hydrostatic levelling across the Straits of Florida would provide basic information concerning the Florida Current, the source of the Gulf Stream. The distance from Miami, Fla., to Cat Cay, B.W.I., is 82 km. ; the greatest intervening depth is about 900 m.

Not only can any two coastal points be joined by hydrostatic levelling, but also any coastal point can be joined with any point on the ocean surface if a tube holding a known liquid starts from the coastal point and has an open end on the bottom near the ocean point. In addition to the pressure p_1 at the coastal point and the atmospheric pressure p_2 at the ocean point, and to the temperature distribution along the tube, the distribution of specific volume below the ocean point must be measured. The path of integration lies from p_1 through the length of tube and vertically upward through the ocean water to p_2.

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[1] Nørlund, N. E., Geodaetisk Instituts Skrifter, København, 3. raekke, **6**, 122 (1945).

[2] Nørlund, N. E., Geodaetisk Instituts Skrifter, København, 3. raekke, **7**, 84 (1946).