THE AIRY INTEGRAL

British Association for the Advancement of Science
Mathematical Tables. Part-Volume B: The Airy Integral, giving Tables of Solutions of the Differential Equation $y'' = xy$. Prepared by J. C. P. Miller. Pp. B.56. (Cambridge: At the University Press, 1946.) 10*s*. net.

THE tabulation in this volume is mainly concerned with the function $Ai(x)$ and its derivative, and before discussing the actual details of the tables, it is instructive to recall the history of the function.

A little over one hundred years ago, in calculations relating to the light-intensity in the neighbourhood of a caustic, Airy was confronted with the integral

$$W = \int_0^\infty \cos \tfrac{1}{2} \pi (w^3 - mw)\, dw,$$

and succeeded in producing a 5-decimal table for values of m ranging between $\pm$ 4, later extended to $\pm$ 5·6.

It can be deduced that W satisfies the differential equation

$$\frac{d^2W}{dm^2} = -\frac{1}{12}\pi^2 m\, W,$$

and hence that the problem of computing it is effectively that of computing the solutions of the second order differential equation

$$\frac{d^2y}{dx^2} = xy.$$

The form of this last equation suggests that its solutions are closely allied to Bessel functions of a certain order, and denoting them by $Ai(x)$ and $Bi(x)$, it may be shown, for example, that when x is real and positive,

$$Ai(-x) = \tfrac{1}{3} x^{\frac{1}{2}} \{ J_{-\frac{1}{3}}(\xi) + J_{\frac{1}{3}}(\xi) \}$$
$$Bi(-x) = (\tfrac{1}{3} x)^{\frac{1}{2}} \{ J_{-\frac{1}{3}}(\xi) - J_{\frac{1}{3}}(\xi) \}$$

where $\xi = \frac{2}{3} x^{3/2}$, with similar results for positive arguments in terms of modified functions.

Recently the demand for tables of the Airy integral has revived; a revival closely connected with the simplicity of the differential equation above, and in virtue of which the British Association, through its tables committee, has prepared this volume (issued as Part-Volume *B*, in conjunction with a volume dealing with the tabulation of Legendre polynomials).

It is provided with a substantial introduction discussing the history of the function, the formulæ and equations involved, and the methods of computing and checking. This occupies some fifteen pages and is followed by tables of $Ai(x)$ and $Ai'(x)$, for the range $x = -20$ to $x = 2$, at intervals of 0·01, the entries being to eight decimal places, with second differences.

Subsidiary tables of the logarithms, zeros and turning values of the functions are also included, and similar results for the Bi functions for the range $-$ 10·0 (0·1) + 2·5 are provided. Finally there are eight pages of tables of auxiliary functions, with definitions and formulæ.

The whole book is beautifully produced and does its authors great credit. It is interesting, in conclusion, to note that Airy's original computations, in spite of the exceedingly laborious work which they involved, were almost entirely devoid of error.

J. H. PEARCE

MAP PROJECTIONS

Map Projections
By George P. Kellaway. Pp. viii + 128. (London: Methuen and Co., Ltd., 1946.) 10*s*. 6*d*. net.

WE welcome a book whose "aim is to provide students of geography with a logical treatment of the commoner map projections". A careful study leaves no doubt of its value as a contribution to the teaching of the subject: the plan is well conceived and skilfully executed.

The earth sphere, latitude and longitude are compressed into four pages, which lead to discussion of the fundamental problem of representation of the earth on flat maps. There is a double classification based on: (1) nature—perspective, geometrical and conventional; (2) construction—on plane, cylinder or cone.

Part 1 begins with polar maps: the order gnomonic, stereographic, orthographic is surprising; there is something to be said for beginning with the sphere as ordinarily seen. Cylindricals follow, and here the discussion of Cassini's is particularly welcome though the construction is not detailed, probably because it is too difficult for those for whom the book is intended. Conicals, ending with Bonne's, complete the geometrical types. Conventionals open with Sanson-Flamsteed. Aitoff receives due notice as an improvement on Mollweide.

The plan of treatment in each case is: (1) mathematical basis; (2) statistical examination of properties at regular latitude intervals; (3) summary of general properties; (4) limitations; (5) construction. The statistical treatment leads to the study of limitations. This is very thoroughly done. Perhaps, after full treatment in one case and a beginning in others, something might, with advantage, have been left for the student to do; on the other hand, much labour has provided some very acceptable reference material.

Part 2 deals with equatorial zenithals and obliques.

The outstanding impression is that the author is deeply concerned, and quite rightly, that the student should realize the importance, in dealing with approximations, of considering the degree of accuracy attained. One would like to think of non-mathematical geographers at school or university being at home with the mathematical work, in these days of numerical trigonometry to School Certificate. To follow the manipulative work would be a severe tax; at worst, the student can take it as read, pay due attention to the statistical data and construct according to instructions. No student can really be said to have studied map projections without constructive work.

The book ends with an admirable discussion of the use of various projections and a collection of representative questions set by university examining boards. Could we not give them a lead in setting questions concerned with travel, and would it not be worth while for examiners to ask a candidate to construct rather than write about projections, at least sometimes?

In conclusion, the reviewer has read the book with real pleasure: he feels no hesitation in recommending its careful study by those seeking a stimulating text-book on the subject or a worthy addition to the geographical library. He congratulates author, publisher and printers on an excellent production.

A. HINCKLEY