

Saddle-Point Methods in the Cascade Theory

ACCORDING to Landau and Rumer<sup>1</sup> and others, the number of electrons of energy greater than  $E$  produced by one fast electron of energy  $E_0$  under a layer  $t$  cascade units thick is given by the formula, neglecting the collision loss and Compton effect:

$$N = \frac{1}{2\pi i} \int_{s_0 - i\infty}^{s_0 + i\infty} \frac{1}{s-1} \left(\frac{E_0}{E}\right)^{s-1} \left\{ \frac{D-\lambda}{\mu-\lambda} e^{-\lambda t} + \frac{\mu-D}{\mu-\lambda} e^{-\mu t} \right\} ds. \quad (1)$$

The integral is usually split into two terms containing the two exponentials, the first is evaluated by the saddle-point method and the second is neglected.

In the following we confirm in a simple way that neglecting the second exponential does not lead to an appreciable error provided  $t \gg 1$ ; at the same time we show that the saddle-point method can still be applied for small values of  $t$ .

In fact, one can write the integrand of (1)

$$\frac{1}{s-1} \left(\frac{E_0}{E}\right)^{s-1} \left\{ \frac{D-\lambda}{\mu-\lambda} e^{-\lambda t} + \frac{\mu-D}{\mu-\lambda} e^{-\mu t} \right\} = e^{\psi(E_0/E, s, t)}; \quad (2)$$

and determine the saddle point by

$$\frac{\partial \psi}{\partial s} = 0 \quad \text{for } s = s_0. \quad (3)$$

One obtains for the integral in the normal way

$$N = \left( \frac{e^{\psi}}{\sqrt{2\pi \frac{\partial^2 \psi}{\partial s^2}}} \right)_{s=s_0} \quad (4)$$

It is immediately clear that the term containing  $e^{-\mu t}$  can be neglected in (3) and (4) without affecting  $N$  appreciably, provided

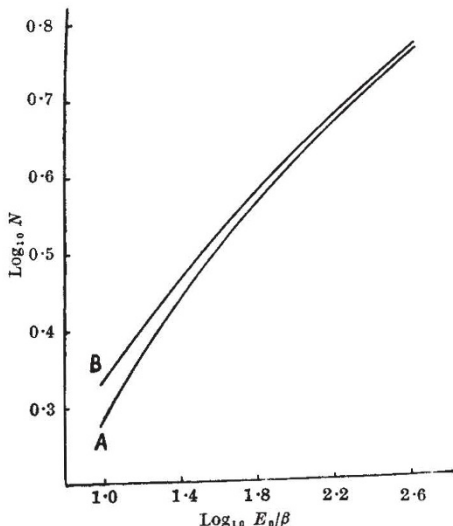
$$\mu t \gg \lambda t \quad \text{for } s = s_0. \quad (5)$$

Thus (3) and (4) reduce to the expressions which are obtained when neglecting the  $e^{-\mu t}$  term to begin with.

We note that (3) and (4) can be used even if  $t$  is small. To take the extreme case for  $t = 0$ , we obtain

$$\frac{e^{\psi}}{\sqrt{2\pi \frac{\partial^2 \psi}{\partial s^2}}} = \frac{e}{\sqrt{2\pi}} = 1.08; \quad (6)$$

whereas the exact value is  $N_{t=0} = 1$ . Thus the error due to the saddle-point method is only 8 per cent. The exp.  $-\mu t$  makes an important contribution to (4).



$E_0$ , ENERGY OF PRIMARY PARTICLE;  $\beta$ , CRITICAL ENERGY OF THE LAYER.

Actually we can apply this method to the expression given by Bhabha and Charkrabarty<sup>2</sup>, including the collision loss. We have calculated  $N(t=1)$  using the first term of Bhabha and Charkrabarty series in powers of  $\beta/E_0$ . The result is given in the accompanying graph, where curve  $B$  is the result obtained by neglecting the exp.  $-\mu t$  term, and curve  $A$  the result by using the exact expression. The energy of the primary particle ranges from  $10\beta$  to  $400\beta$  and the corresponding difference between the two curves from 15 to 2 per cent.

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<sup>1</sup> Landau and Rumer, *Proc. Roy. Soc., A*, **166**, 213 (1938). Serber, *Phys. Rev.*, **54**, 317 (1938).  
<sup>2</sup> Bhabha and Charkrabarty, *Proc. Roy. Soc., A*, **186**, 267 (1943).

Elastic Constants of Isotropic Media

USING ultrasonic waves of continuously varying frequency, it was recently observed<sup>1</sup> in this laboratory that characteristic thickness shear modes could be transmitted through crystal plates and communicated to liquids in the form of consequential longitudinal strains. Further work on these lines has revealed that such modes can be similarly transmitted through isotropic media. This offers a new method of obtaining the entire frequency spectrum of a given plate, and therefore its elastic constants, even when it is available only as a small fragment. Using a tourmaline wedge with a frequency range of 3-12 megacycles per second, plates of glass, steel and brass have been studied. Values of Young's modulus  $Y$  and the rigidity modulus  $n$  obtained are compared with the usual static results in the accompanying table.

| Material | Smallest size used | Authors' results |      | Static values |         |
|----------|--------------------|------------------|------|---------------|---------|
|          |                    | Y                | n    | Y             | n       |
| Glass    | 7 x 7 x 0.93 mm.   | 4.80             | 1.76 | 4.60          | 1.80    |
| Steel    | 8 x 8 x 0.64 "     | 21.71            | 8.40 | 19-20         | 7.7-8.3 |
| Brass    | 10 x 6 x 0.58 "    | 9.77             | 3.50 | 9.7-10.2      | 3.5     |

The static values for glass were also determined for the same sample, and those for steel and brass have been taken from the literature. The unit is  $10^{11}$  dynes per sq. cm. and the dimensions of the smallest plate used in each case are given although plates of larger size have also been worked upon.

The appearance in this case of shear modes as corresponding longitudinal waves in the adjoining liquid is presumably due to a coupling effect arising from the finite size of the plates. This view is supported by our observation that such modes gain in intensity and become prominent as the plate is made smaller.

A fuller report will be published elsewhere.

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<sup>1</sup> Bhagavantam, S., and Bhimasenachar, J., *Proc. Ind. Acad. Sci.*, **20**, 298 (1944); *Nature*, **156**, 23 (1945).

Thermodynamics and the Logarithmic Temperature Scale

THE explanation of the reason for the existence of the Second Law of Thermodynamics and the meaning of entropy has always been inadequate, but the use of the logarithmic temperature scale<sup>1</sup> helps to simplify the fundamental concepts.

The distribution of gaseous molecular velocities has been studied experimentally<sup>2</sup>, and is of the order shown in the accompanying figure where the distribution for two temperatures represented on the logarithmic scale as  $D_1$  and  $D_2$  are shown and which overlap. The nearer these two temperatures are to each other, the larger will be the number of molecules which are common to both temperatures. Although the common molecules are available for contribution to the heat energy at both temperatures, they will not be available for doing work. As the temperatures recede from each other more molecules are available for doing work.

A consideration of work and heat energy shows that work energy is defined by the use of a potential factor and a quantity factor,

