in tables, though (b) seems not to fit quite so well as the assumption $\psi \equiv 1$ (logarithmic law).

	Function	$\lvert \psi - 1 \rvert_{\max.}$ ($A = 10$)
(a)	$\frac{2}{\sqrt{2\pi a}} e^{-x^2/2a^2}$	0·33
(b)	$\frac{2a}{\pi (a^2 + x^2)}$	0·0557
(c)	$\frac{4a}{\pi^2} \cdot \frac{\ln (x/a)}{x^2 - a^2}$	0·00152
(d)	$\frac{1}{a} e^{-x/a}$	0·115
(e)	$\frac{a}{(a + x)^2}$	0·0065

For $A = 100$ the values of $\lvert \psi - 1 \rvert_{\max.}$ become much larger; 0·24 for (e) and 0·11 for (c); thus (c) alone is not clearly inconsistent with the scanty data from counts on this basis.

Since the accuracy of the approximation $\psi \cong 1$ is not improved by increasing the scale factor α, we are led to try mixing together distributions of type $\frac{1}{\alpha} f\left(\frac{x}{\alpha}\right)$ with various values of α. If we assume that in the final mixture a fraction $\frac{1}{\beta} g\left(\frac{\alpha}{\beta}\right) d\alpha$ of the entries comes from distributions with values of α between α and $d\alpha$, then the fraction between x and $x + dx$ will be $\frac{1}{\beta} h\left(\frac{x}{\beta}\right) dx$, with

$$\frac{1}{\beta} h\left(\frac{x}{\beta}\right) = \int_0^\infty \frac{1}{\alpha} f\left(\frac{x}{\alpha}\right) \frac{1}{\beta} g\left(\frac{\alpha}{\beta}\right) d\alpha. \quad . \quad . \quad . \quad (9)$$

If, in particular, $g(x) \equiv f(x)$, then we shall call h the 'iterate' of f. In our table the Cauchy function (b) is the iterate of the Gaussian (a), (c) is the iterate of (b), and (e) is the iterate of (d). It is evident from the table that iteration does rapidly decrease the value of $\lvert \psi - 1 \rvert_{\max.}$.

The relation (9) of iteration corresponds to the following relation between the distributions $\psi(q)$ connected with f, g, h by formulæ of the type of (4):

$$\psi^{(h)}(q) = \int_0^1 dr \cdot \psi^{(g)}(r) \psi^{(f)}(q - r). \quad . \quad . \quad . \quad . \quad (10)$$

If we represent each distribution $\psi(q)$ by a Fourier series,

$$\psi(q) = \sum_{-\infty}^{\infty} a_n e^{2\pi i n q}, \; a_{-n} = a_n^*, \quad . \quad . \quad . \quad . \quad . \quad (11)$$

then (10) gives

$$a_n^{(h)} = a_n^{(g)} \cdot a_n^{(f)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

In particular, iteration of a distribution function corresponds to squaring each Fourier coefficient a_n. One can show readily that since $\psi(q)$ is nowhere negative, $a_n < 1$ for all n; thus it is evident that iteration repeated a sufficient number of times will make $\lvert \psi - 1 \rvert_{\max.}$ arbitrarily small. If one assumes that only one harmonic exists—that is, $a_n = 0$, $n \neq n_0$—then it can easily be seen that the value of $\lvert \psi - 1 \rvert_{\max.}$ for the iterate is half the square of the value for the original function. This relation holds within about 2 per cent for the cases given in the table; the appearance of the computed curves of $\psi(q)$ shows that only the first harmonic is important in the cases in hand.

The fact that the close agreement between the trapezoidal sum and the integral in cases like (c) and (e) is not at all to be attributed to broadness of the distribution is seen very forcibly in the process of computing the sums numerically. In all cases considered, at least two thirds of the sum comes from the largest two terms, and at least seven eighths from the largest three. For three of the sets of ordinates computed for case (c) numerical integrations were performed by Weddle's rule, and the fractional errors found were 0·00612, 0·01224 and 0·01905—four, eight and twelve times the maximum error of the trapezoidal rule for ordinates at the given spacing.

The remarkable affinity between certain functions such as (c) and (e) and the trapezoidal rule enables us to write approximate relations of a peculiar sort. For example, from (e) we get

$$\frac{1}{\ln Z} \cong \frac{1}{4} + 2 \sum_{k=1}^{\infty} \frac{Z^k}{(1 + Z^k)^2}, \quad . \quad . \quad . \quad . \quad . \quad (13)$$

for Z real and greater than unity but not a large number. This is never exactly true, but the error becomes small very rapidly as Z is decreased. For $Z = 10$, the error is 2/3 per cent; for $Z = 4$, it is 0·004 per cent; and for $Z = 2$, it is not more than about 10^{-7} per cent and cannot be determined readily by using an ordinary ten-digit computing machine.

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Hissing Sounds Heard During the Flight of Fireballs

MANY responsible eye-witnesses, in their descriptions of fireballs, have emphatically stated that they have occasionally heard a peculiar hissing sound *simultaneously* with the flight of a meteor. From personal observation, I can also testify to the validity of these statements. Fireball literature is full of such accounts. Three recent cases (connected with fireballs seen by a number of competent observers at Hyderabad, on October 13, 1936, on March 25, 1944, and on August 6, 1944, respectively) have placed the matter beyond any doubt whatever.

The obvious difficulty is about the simultaneity of the light and sound phenomena noticed by observers fifty to a hundred miles distant from the meteor. But it must be remembered that the fireball rushes through the upper atmosphere with parabolic speed (about 26 miles per second); its duration of visible flight is generally 6–8 seconds. Assuming its height to be roughly 75 miles, *matter* from a *friable aerolite* can issue in a regular stream along its entire path, into the lower atmosphere, with velocity large enough to bring it in the vicinity of an observer while the meteor is still in sight. For the height assumed, four or five seconds may suffice (even allowing for air resistance) for the *matter* from the meteor to reach the air in the neighbourhood of the observer, and thus give rise to sounds variously described as like the swish of a whip, the hissing noticed while a cutler sharpens a knife on a grindstone, or a hot iron being plunged into cold water.

A shower of fine sand beating against the leaves of trees was noticed immediately after the apparition of the fireball of October 13, 1936, described in detail in *Science and Culture*, Calcutta, 2, No. 5, 273 (1936).

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