James Gregory (1638-1675)*

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IN the early seventeenth century, two brothers, Alexander and David Anderson of Aberdeen, rose to mathematical distinction. Alexander, who went to Paris as professor of mathematics, became the friend and expositor of Vieta, while David, commonly called 'Davie-do-a'-thing', became the Archimedes of Aberdeen, constructed the spire of St. Nicholas Church, and removed 'Knock Maitland', a dangerous submerged rock, from the harbour by harnessing it to the tide. His daughter Janet married the Rev. John Gregory, of Drumoak on Deeside, twelve miles from Aberdeen, and thereafter for two hundred years their descendants occupied the chairs of either mathematics, medicine or philosophy in an unbroken sequence at Scottish universities, always with vitality and distinction.

The greatest of this remarkable family was James, the third son of John and Janet Gregory, who gained renown for the invention of the reflecting telescope, but was also a mathematician whose genius is outstanding even among the giants of the age of Newton. James was born in 1638, the year when Descartes gave analytical geometry to the world and Galileo his laws of motion; he died prematurely in 1675, of a sudden illness accompanied by blindness which befell him as he was observing the planet Jupiter in company with his students at Edinburgh. He received his first lessons in mathematics from his mother; he eagerly mastered Euclid's "Elements" given to him by his elder brother David at the age of thirteen or fourteen; he went to the Grammar School and later to Marischal College at Aberdeen, where he graduated.

By the age of twenty-four years, James Gregory had written his "Optica Promota", which contains an elegant and geometrically accurate account of mirrors and lenses, beginning with the re-discovery of the sine law (of Snellius and Descartes), dealing with elliptical and hyperbolic surfaces and ending with a description of a reflecting telescope. An astronomical appendix also suggests the importance of observing the transits of Venus and Mercury for calculating parallax. In 1663, Gregory went to London and the book was published. He met Collins and Hooke, who put him in touch with the celebrated optician Reive; Reive tried to make such a telescope, but the resulting mirrors were a failure and the attempt was abandoned. But Hooke, that prince of experimenters,

persevered, and in February 1674 presented the first Gregorian telescope to the Royal Society, and the same form was universally employed in the eighteenth century.

Gregory spent the next three years in Italy, chiefly at Padua, where the great Galileo had taught. There Gregory printed, in 1667, his "Vera Circuli et Hyperbolæ Quadratura", wherein he showed how to find the area of the sector of a conic by means of convergent polygons, and actually attempted to prove the transcendence of the ratios nowadays denoted by π and e . (He considered π and also the number $2.3025850929940456 \dots$ which is $\log_e 10$; this he calculated correctly to seventeen figures.) The book lays the foundations of hyperbolic sines and cosines, as well as the concepts of convergence and algebraic invariance under transformations.

In 1668, Gregory returned to England with a third book, the "Geometriæ Pars Universalis", which gives a systematic *geometrical* account of the calculus, based on the work of Cavalieri and Fermat. It contains the earliest proof of the fundamental theorem—that the method of tangents (differentiation) and of quadratures (integration) are converses of each other. It also shows how to transform $\int y dx$ to $\int z dy$, where $z dy/dx = y$. This book profoundly influenced Barrow, who published his "Geometriæ Lectiones" in 1670.

Gregory remained in London until the autumn of 1668 with Collins, who egged him on to solve the unproved theorems of the day. As a result he produced a booklet, the "Exercitationes", containing a remarkable method of reversing a power series $f(x)$ in the form $x = \sum c_n f(x/2_n)$, together with rigid proof of Mercator's theorem on the expansion of $\log(1+x)$. By integrating $\sec x$ and $\tan x$ Gregory cleared up the mystery of the rumb spiral, explaining why Wright's "Nautical Tables" of 1599 actually presented a set of logarithms, of which the author was unaware.

In 1668 Gregory was appointed first professor of mathematics in St. Andrews, where he resided until 1674. Partly owing to a conflict with Huygens over the "Vera Quadratura" and partly through hearing of Newton's success in the "Analytics", Gregory never again published his mathematical work, beyond adding one short note to a vehement little book on the "New Art of Weighing Vanity", written by a colleague, William Sanders, a regent at St. Leonard's College, St. Andrews; this book was directed against George Sinclair, a

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retired professor of natural philosophy at Glasgow, who had quarrelled with Sanders. The note contains the expansion of an elliptic integral in a doubly infinite series and solves the finite motion of a circular pendulum. In 1674 Gregory was appointed to the chair of mathematics in Edinburgh, which he only held for a year before he died. He married in 1669 Mary, daughter of George Jamesone the painter, and widow of Peter Burnet, of Aberdeen, and they had two daughters and a son, James, afterwards professor of physic in King's College, Aberdeen (d. 1731).

A long correspondence (1668-1675) between Gregory and Collins has happily been preserved. Gregory's letters were printed in Rigaud's "Correspondence of Scientific Men of the 17th Century". Collins's letters, which are now in the University Library at St. Andrews, have never yet been published, but the present tercentenary year affords an appropriate opportunity for doing so,

particularly as they contain—on the blank spaces of the letters—some fifty mathematical notes of James Gregory in his own hand, revealing either the methods employed in such results as he sent to Collins by letter, or in still newer work which never saw the light. Careful scrutiny, begun in 1932, has unravelled their contents, and shows that Gregory actually anticipated Taylor, by forty-five years, in his famous expansion theorem, that Gregory could integrate $\sin^m \theta$, and had gone deeply into the theory of rational right-angled triangles and Diophantine quadratic equations. It is now possible to judge how slight was his mathematical contact with Newton; also how Barrow and Gregory influenced one another. It is noteworthy that Gregory probably never even heard of the name of Leibniz. These discoveries among the old letters go far to explain the fact that in his day Gregory was held to be second only to Newton.

Vaccination of Cattle against Foot-and-Mouth Disease

THE devastating epizootic of foot-and-mouth disease which has raged over Western and Central Europe amongst cattle, sheep and pigs since last autumn has by no means come to an end, though there have been some signs of local or temporary abatement in those areas first invaded in Holland, Belgium and parts of France and Germany. Sweden, Norway and Denmark have suffered little, and are always ready to stop early invasion by slaughter. In Great Britain there have been exceptionally severe and numerous outbreaks, but the number of farms affected and of animals infected or lost by slaughter has not approached that of the Continental calamity. On June 1 last, the number of farms infected in Germany was officially reported as 42,306.

Persistent research work in Germany has been carried out for forty years almost continuously, and since the Great War by Waldmann on the island of Riems; in France much valuable work was also done by Vallée at Alfort. In Great Britain research has been in progress since 1924, but though knowledge of the disease has increased, it has only been possible to increase the means of control in special circumstances and to reduce the spread of infection. Experimental stations have also been set up in recent years in Holland and Denmark.

The chief aim of investigation has been to find a method of protective vaccination, and many attempts have been made to devise a safe vaccine made from the virus, attenuated by biological, chemical or physical treatment, or made safe by the manner of its administration.

Inactivation of the virus by formalin has until recently appeared to be the most hopeful method, and has perhaps never been tried in the field with sufficient care and persistence. During the last few years, a technique of adsorption of virus with aluminium hydroxide has been very carefully worked out with great detail by Schmidt and his colleagues in Denmark on guinea pigs, and very promising results have been published. He attached great importance to the administration of the adsorbate-vaccine subcutaneously, for by other routes it was capable of producing infection. The results of vaccination given by Schmidt closely resemble those obtained with formalinized vaccine, but the adsorbate appears to be less safe and to give a somewhat higher degree of protection, as might be expected from a less completely inactivated virus. Waldmann maintains that Schmidt's vaccine is too dangerous, and he does not accept the suggestion that the active virus can only be released so slowly from the adsorbate as to be safe though still capable of giving active immunity.

A modification of Schmidt's vaccine has recently been used by Waldmann in Germany with additional safeguards to ensure its inability to produce the disease and spread infection. It was first tested on cattle at the Riems station with good results. The accessory chemical or physical means of attenuation devised by Waldmann have not yet been published, though an account of the vaccination of more than 40,000 cattle with this preparation has appeared in the *Berliner tierärztlicher Wochenschrift*, Nos. 22 and 24 (1938).