cane and beet-sugar production, he treats each major producing area in turn, stressing the variations in unit-yields from one area to another. The huge expansion in consumption which is considered possible could largely be met by increasing the yields in Cuba (2-3 tons per acre) or India to those of Java which have risen from  $1 \cdot 1$  to  $6 \cdot 5$  tons in the last century. The Indian statistics cease with 1932-33, but the recent expansion is predicted. It is considered that if protection were removed, beet-sugar would practically disappear. L. D. S.

### Europe: a Regional Geography

By Margaret Reid Shackleton. (University Geographical Series.) Pp. xvi+430. (London, New York and Toronto : Longmans, Green and Co., Ltd., 1934.) 15s. net.

BEHIND the freshness of the writing and the sometimes almost casual treatment of a wide field there is evidence of extensive reading, a sympathetic understanding and not infrequently of an acute perception reminiscent of L. W. Lyde at his best. This is not a school book, nor is it an adequate university textbook, but it may be cordially recommended to all who have been accustomed to, and have perhaps tired of, the formal presentation of the facts of European geography and who seek a broader view in the understanding of current problems. Britain is excluded from consideration.

## Mathematics

#### The Differential Calculus

By Theodore Chaundy. Pp. xiv+460. (Oxford : Clarendon Press; London : Oxford University Press, 1935.) 35s. net.

THERE has long been a need for a modern English textbook on the advanced parts of differential and integral calculus. The older books were comprehensive in scope, but deficient in rigour; the newer ones give a careful discussion of limits and continuity, but they ignore such topics as Jacobians and maxima and minima of functions of two independent variables. Many years ago, Mr. Chaundy started to write a differential calculus which should combine the English plan with Continental rigour, but in the course of time it has developed into something rather difficult to classify.

The book is intended to be a complete and selfcontained treatment of the differential calculus so far as it concerns the real variable, excluding applications to geometry and mechanics. It would have been more convenient to students if the author had abandoned his attempt to be self-contained, an ideal not fully attained, as on p. 14 a rather difficult theorem on limits is quoted from Bromwich's "Infinite Series". The contents of Hardy's "Pure Mathematics" might reasonably have been assumed to be known to every student of higher mathematics ; by taking this as a starting point, Mr. Chaundy could have made his book much shorter and cheaper.

There are fourteen chapters, dealing respectively with functional dependence, the continuous function (two), the derived function, higher derivatives, partial differentiation, indeterminate forms, analytic functions, maxima and minima (two), implicit functions, Jacobians and Hessians, differential operators, and expansions in power series. Many parts are of great interest, and some of these contain information difficult to find elsewhere. New terms are introduced, such as "vanishingly uniform convergence", and "umbral derivative", and there are several variations from the orthodox notation, some of which seem to be improvements. There are about four hundred carefully chosen examples, and a detailed index.

#### Lectures on Matrices

By J. H. M. Wedderburn. (American Mathematical Society Colloquium Publications, Vol. 17.) Pp. vii+200. (New York: American Mathematical Society; Cambridge: Bowes and Bowes, 1934.) 3 dollars.

THE calculus of matrices has had a curious history. It was first used by Hamilton in 1853 under the name of "Linear and Vector Functions". Cayley used the term matrix in 1854, and developed the basic notions of the theory in 1858 without recognising the relation of his work to that of Hamilton. The algebra of matrices was rediscovered by Laguerre in 1867 and by Frobenius in 1878. Sylvester wrote a large number of papers on matrices, especially in the years 1882-84. After this, although the subject was never entirely dropped, it certainly seemed for many years to be regarded as of little interest to mathematicians in general. Then suddenly came a change. In 1925 Heisenberg's quantum mechanics created a widespread interest in matrices among physicists and applied mathematicians. It is significant that in the bibliography given at the end of the book under review, the average number of entries for 1853-1924 is  $5 \cdot 3$  a year, rising to  $18 \cdot 3$  for the years 1925-33, although the basis of selection has been narrowed for the later years.

Dr. Wedderburn's book is founded on lectures given at Princeton University at various times since 1920. It contains ten chapters, dealing respectively with matrices and vectors, the characteristic equation, invariant factors, vector polynomials, compound matrices, Hermitian matrices, commutative matrices, functions of matrices, automorphic transformations, and linear associative algebras. The subject is treated from the point of view of pure mathematics, without any explicit reference to physics, but some of the topics may easily be recognised as those which are of great importance in quantum mechanics.

# Generalized Hypergeometric Series

By Dr. W. N. Bailey. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 32.) Pp. vi+108. (Cambridge : At the University Press, 1935.) 6s. 6d. net.

THE ordinary hypergeometric series, discussed by Gauss about a century ago, is well known; it is a function of three parameters, of which two occur in the numerator and one in the denominator. In the