The Waves of an Electron.¹

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THE history of the newer physics, as it may be called, is largely the history of the very successful attempts which have been made to explain the facts of the physical universe in terms of electricity. In particular, the natural units of electricity, the electron first isolated as the carrier of the negative charge in the cathode rays, and the more massive proton with its equal positive charge, have been the bricks out of which physicists have tried to build model atoms. But until recently the only properties which could be attributed with certainty to the electron were its mass, 9×10^{-28} gm., and its charge, -4.77×10^{-10} electrostatic units. It is indeed possible to calculate a 'size' on the assumption that it follows the same laws as a charged metal sphere in a laboratory, but the value found, 3.7×10^{-13} , is neither confirmed nor contradicted by any experimental evidence.

To show why we are now obliged to ascribe much more complicated properties to the electron, I must make a digression into optics. It is well known that there is a number of phenomena which are examples of wave-motion in what used to be called the ether, though it seems now more fashionable to call it space. Such are wireless waves, light, ultra-violet radiation, X-rays. They differ only in wave-length, and it is convenient to refer to them all as light.

Now at the beginning of the century, the wave theory of light was one of the most firmly established parts of physics. It is true that for many purposes light was treated as consisting of rays, that is, effects travelling in a straight line, but this was merely an approximation used for convenience in calculation, and not regarded as of any great physical significance. The evidence for waves (interference and diffraction) was, and remains, overwhelming. Gradually, however, a series of experimental and theoretical results accumulated which showed that this was not the whole story, and in particular that light could communicate an amount of energy to an electron which only depended on the *kind* of light and not at all on the *intensity*. If light consists of continuous waves this seems almost impossible; but it is just what we should expect if, as Newton firmly held, light is due to a stream of corpuscles. In such a case, if one of these hit an electron, it would give to it an energy which would be quite independent of the number of other corpuscles present in the stream, that is, of the intensity of the light.

Louis de Broglie has hit on a brilliant synthesis of the two views. Put very briefly, it comes to saying that light is indeed corpuscular, but that the corpuscles are accompanied and guided by waves. Now the bearing of this on our subject is as follows: de Broglie's theory is quite a general one, and should apply to *any* sort of corpuscular

motion; in particular electrons should be accompanied by waves which determine their motion. It would seem at first sight that this would involve a contradiction with what is already known about the motion of electrons. For example, waves spread after passing through a small aperture, while electrons can be concentrated into a fine beam by a tube or series of slits. But it is all a question of wave-length. Ordinary visible light only spreads very slightly round obstacles because its wave-length is small compared with the size of ordinary objects. Wireless waves bend much because their wave-length is large.

There is, in fact, a relation pointed out more than a century ago by Hamilton between optics and dynamics. The laws of dynamics in their most general expression, the principle of action, are mathematically identical with Fermat's principle, which is the most general statement of optics on the ray theory. Now in optics, rays are an approximation to the wave theory, more and more nearly true the smaller the wave-length; perhaps

FIG. 1.—Rings produced by a gold film.

the accepted mechanics of Newton and Hamilton is also only an approximation, true in practice because the wave-lengths concerned are very small? If so, we must develop a new mechanics to deal with cases when the wave-length is not of negligible size.

De Broglie's results are as follows: For any moving particle the wave-length $\lambda = h/(\text{momentum})$ where h is Planck's constant $= 6.58 \times 10^{-27}$. This in fact comes out negligibly small for any ordinary body. Even for an electron moving with the moderate energy of 150 volts it is 10^{-8} cm., or about that of a soft X-ray, while for cathode rays of, say, 15,000 volts, it is 10^{-9} cm.

Still, Sir William Bragg and his son have shown us how to measure the wave-lengths of X-rays, small though they are, and the electron waves may yield to a similar treatment.

We must first consider some general properties of these waves. According to de Broglie the speed of the waves, V , is c^2/v , where v is the speed of the particle, and c the velocity of light. They are thus faster than light. This result would be contrary to relativity if the waves carried energy, and so could transmit a signal, but as the energy is supposed associated only with the *particle*, this objection does not come in; indeed, on the classical

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optical theory, cases are known of metals, such as sodium, having a refractive index less than unity, which implies a wave velocity greater than c . From the above two equations, taking account of the variation of mass with speed, $V^2 = c^2 + \frac{m_0^2 \lambda^2}{h^2 c^4}$, and so depends on λ . Thus there is *dispersion*, in the optical sense, for these electron waves even in free space.

The late Lord Rayleigh showed that in such a case, besides the velocity of the independent waves, there is what he called a *group velocity*, which is the speed with which the pattern formed by the superposition of several waves of slightly different wave-length moves along. Now an application of his formula shows that in this case the group velocity is precisely v , the speed of the electron, so that the electron carries its *group* with it—or the group carries the electron—though the individual waves composing the group are perpetually shooting through from behind.

Again, since an electron is charged, its motion is affected by an electric or a magnetic field. On the older view of an electron this was simply a consequence of the laws of electricity. On the

tions. It forms, in fact, the three-dimensional analogue of the ordinary optical ruled diffraction grating.

The idea of using a crystal in this way has been carried out by Davisson and Germer in America, and by Reid and myself in Great Britain. If a beam of X-rays is passed through a single crystal and allowed to fall on a photographic plate, the developed picture is a series of spots the position of which depends on the structure of the crystal. If it passes through a powder formed of a large number of very small crystals arranged at random, the spots due to each tiny crystal join together to form a series of concentric rings, the relative sizes of which depend on the crystal structure, the absolute size of the whole pattern being proportional to the wave-length of the X-rays. Both these methods, or their equivalent, have of course been extensively used in the investigation of crystal structure by X-rays.

The apparatus used by Reid and myself is a direct application of this method to the study of electrons. A narrow beam of cathode rays passes normally through a very thin film of metal by which it is scattered or diffracted. The scattered

RELATIVE DIAMETER OF RINGS.

Metal . .	$\sqrt{3}$	$\sqrt{4}$	$\sqrt{8}$	$\left\{ \frac{\sqrt{11}}{\sqrt{12}} \right\}$	$\sqrt{16}$	$\left\{ \frac{\sqrt{19}}{\sqrt{20}} \right\}$	$\sqrt{24}$	$\sqrt{27}$	$\left\{ \frac{\sqrt{35}}{\sqrt{36}} \right\}$
Gold . .	$\sqrt{2.93}$ $\sqrt{3.05}$ $\sqrt{2.96}$	$\sqrt{4.02}$ $\sqrt{4.05}$ $\sqrt{4.08}$	$\sqrt{8.00}$ $\sqrt{8.00}$ $\sqrt{8.00}$	$\sqrt{11.2}$ $\sqrt{11.4}$ $\sqrt{11.2}$		$\sqrt{19.8}$ $\sqrt{19.1}$ $\sqrt{19.5}$		$\sqrt{22.7}$	$\sqrt{26.6}$ $\sqrt{26.8}$ $\sqrt{35.4}$
Aluminium		$\sqrt{4.00}$ $\sqrt{3.98}$ $\sqrt{4.02}$	$\sqrt{8.00}$ $\sqrt{8.00}$ $\sqrt{8.00}$	$\sqrt{10.9}$ $\sqrt{11.0}$ $\sqrt{10.9}$	$\sqrt{16.5}$ $\sqrt{15.7}$ $\sqrt{15.9}$	$\sqrt{20.4}$ $\sqrt{21.2}$ $\sqrt{19.4}$		$\sqrt{27.0}$ $\sqrt{27.8}$ $\sqrt{26.8}$	$\sqrt{35.0}$
Platinum	$\sqrt{3.00}$ $\sqrt{3.07}$ $\sqrt{3.04}$	$\sqrt{4.00}$ $\sqrt{4.01}$ $\sqrt{4.03}$	$\sqrt{8.00}$ $\sqrt{8.00}$ $\sqrt{8.00}$	$\sqrt{10.9}$ $\sqrt{11.3}$ $\sqrt{11.2}$	$\sqrt{15.3}$ $\sqrt{16.4}$	$\sqrt{18.4}$ $\sqrt{19.0}$	$\sqrt{24.0}$ $\sqrt{23.4}$	$\sqrt{26.5}$ $\sqrt{27.1}$	$\sqrt{35.0}$

new view the electron goes where its wave takes it. Somehow, the only possible paths are those along which the wave is propagated. On this view the change in motion of the electron is analogous to the change in direction of a ray of light when it enters a refracting medium. In fact, by choosing the field in an appropriate manner, we can imitate the effect on light of a corresponding piece of glass. For example, we can make a field equivalent to an ordinary convergent lens and bring electrons to a focus.

Of course, this is only a translation between one view and the other. To test the new theory we must use something which has a regular structure comparable in size with the supposed wave-length. Here the wide experience obtained with X-rays comes to our aid, and the natural regularities of a crystal give what is needed. The regularly spaced atoms of a crystal each scatter the waves, since they scatter the electrons. Such a system of regularly arranged scattering points produces, as is well known, a peculiar effect on a simple wave, which is deflected into a few definite direc-

rays diverge till they reach a photographic plate parallel to the thin film and about 30 cm. away, where their impact is recorded. It is necessary to use very thin films because electrons have vastly less penetrating power than X-rays, and they must not only penetrate the film, but also be so little affected by it that the scattering is what is technically called 'single.' In practice this means that only a small minority should be appreciably scattered at all. The films we used were only a few millionths of a centimetre thick; in fact, it is no use trying a metal film unless it is thin enough to be transparent. Some of the metals give uniform rings (Fig. 1), others spots arranged round rings. Both can be paralleled in analogous results with X-rays.

In comparing theory with experiments we have three separate checks. First, the relative sizes of the rings must conform to the known crystal structure. For the metals I used they go as the square roots of certain integers. In fact, the first few rings should be in the ratio $\sqrt{3} : \sqrt{4} : \sqrt{8} : \sqrt{11} : \sqrt{12} : \sqrt{16} : \sqrt{19} : \sqrt{20}$. The above table shows the extent of the agreement.

The rings corresponding to $\sqrt{11}$ and $\sqrt{12}$, $\sqrt{19}$ and $\sqrt{20}$, and $\sqrt{35}$ and $\sqrt{36}$ were generally too close together to be separated.

Secondly, if de Broglie's theory is true, the wave-length is inversely as the momentum, that is, apart from a small relativity correction, the product of the diameter of any one ring into the square root of the measured energy of the rays should be constant.

Aluminium.		Gold.		Platinum.	
Voltage.	$D\sqrt{P}$.	Voltage.	$D\sqrt{P}$.	Voltage.	$D\sqrt{P}$.
21,800	(359)	21,000	(344)	18,200	(378)
34,500	385	33,700	377	25,500	395
45,000	385	44,000	377	34,500	420
57,600	395	55,000	379	40,000	401
64,000	385	58,000	376	45,000	413
Means	387		377		407

Celluloid.			
Voltage.	$D\sqrt{P}$.	Voltage.	$D\sqrt{P}$.
9,800	185	23,200	193
11,500	175	30,500	186
16,100	189	36,000	193
16,800	191	42,500	189
21,000	190	50,000	195

Apart from the numbers in brackets, which corresponded to very faint rings, the agreement is satisfactory.

Thirdly, the actual size of the rings is calculable from the assumed wave-length, and the known spacing of atoms in the crystal. The best way of showing the agreement is to work back, calculating the crystal spacing from the experiments and theory, and comparing with that found by X-rays.

SIZE OF CRYSTAL UNIT.

	Aluminium.	Gold.	Platinum.
X-ray measurements . .	4.046×10^{-8}	4.06×10^{-8}	3.91×10^{-8}
Cathode ray measurements	4.06×10^{-8} 4.00×10^{-8}	4.18×10^{-8}	3.88×10^{-8} 3.89×10^{-8}

The bracketed measurements were made with different apparatuses.

Finally, in a few cases the individual spots can be used. This is when the small area struck by the rays happens to consist wholly or mainly of a single crystal. In such a case we can find the orientation of the crystal from some of the spots and see if the others check, as in fact they are found to do.

I think that this agreement may be regarded as establishing the essential truth of the theory.

There remain great difficulties of interpretation. What are these waves? Are they another name

for the electron itself? How many waves in series does an electron have? What is the relation between the electron waves and those of light and X-rays? Some of these questions I should like very briefly to discuss, but we now leave the sure foothold of experiment for the dangerous but fascinating paths traced by the mathematicians among the quicksands of metaphysics. First, as to the length of the train of waves. Here we have a little direct evidence. The sharpness of the rings depends, among other things, on this length. A short train of waves results in blurred rings. If we suppose the whole width of the rings due to this, I calculate from the fact that it is sometimes possible to separate the $\sqrt{11}$ and $\sqrt{12}$ rings that there must be at least 48 ordered waves in the series. Probably there are many more, as several other causes tend to widen the rings. But this may only apply to my particular experimental arrangement.

Theoretically, it seems probable that an exact knowledge of the speed of an electron implies that it has an infinite wave-train associated with it, just as ideally monochromatic light implies an infinite train of light waves. Since there cannot actually be an infinite wave-train, this means that an electron's speed can never be exactly known, which in one sense is not surprising, for all measurement is approximate, and it can be shown that the conditions under which the velocity of an electron could be found with great accuracy are precisely those in which a long train of waves could accompany it. But suppose we have a short train of waves. The velocity is uncertain. Is this because there are really many electrons concerned and they have different velocities? In most of the experiments one could devise there would be, but I do not think this covers all cases.

Imagine a small speck of radio-active material which emits electrons as fast β -rays. If we make it small enough the particles may follow each other at intervals of minutes, or days, or years. It seems impossible to suppose that these electrons have any connexion with each other. They come from different atoms at widely different times. One is forced to the conclusion that each must have its own separate wave system. But this wave train must be finite, probably in fact it is rather short, and one reaches the surprising conclusion that the electron *has* no definite velocity, even though it may be moving in force-free space. This raises obvious difficulties about energy which I do not pretend to be able to solve, but the work of Ellis and Worster on radium *E* seems to show that there is an uncertainty of this kind, which is in fact of about the right amount.

As regards the relation between electron waves and light waves: they are certainly not the same, even when their wave-lengths are equal. They go at different speeds: one is refracted by electric and magnetic fields, while the other is not; their penetrating powers in ordinary matter are widely different. If they are the actual motion of an ether, it must differ in some way in the two cases.

My father, Sir J. J. Thomson, has shown that the relation between wave-length and velocity of

the electron waves is precisely that which holds for light waves in a medium containing free electric charges. The properties of such a medium have recently become of great importance, for the possibility of long-distance wireless has been shown to be due to the existence of a region with these properties in the upper atmosphere. It is tempting to regard this agreement as something more than a mere coincidence, and to consider the waves of the electron as waves in an ether modified by the presence of the tubes of force, which since the days of Faraday many physicists have regarded as having a real existence round an electric charge. Thus the electron would provide, not indeed its own ether, but the necessary modification of that ether.

I should like also to suggest another possibility. Personally, I see no necessity for there to be any vibration of a material or quasi-material object. The essence of a wave is the transference of a *state* according to certain laws. The state may, or may not, be one of motion. On this view the function of the waves is to produce conditions at the electron which oblige the latter to move along the instantaneous position of the wave-normal (in the absence of a magnetic field), and along another, but determinate, direction in a magnetic field. But though it seems possible to regard the motion of the electron as determined by the wave in its immediate neighbourhood, it must never be forgotten that the direction of each part of a wave-front depends on the history of the whole wave-front. Any circumstance that influences one part of a wave influences the whole, and this is of the essence of wave motion. The electron is influenced by conditions at a distance, but indirectly through its waves. Thus in our experiments the electron is influenced not merely by the nearest atoms of the crystal, but also by the whole of the crystal which is exposed to the beam. The *width* of the wave-front must be taken as the cross-section of the tube which limits the beam of electrons.

Although the whole wave-front, in theory, affects the electron, all parts do not affect it equally. Thus, suppose the width of the beam were diminished by gradually reducing the size of the tube, apart from the obvious diminution in intensity, the only effect would be a diminution in resolving power due to the reduction in the number of grating elements. When the resolving power is large a considerable

change, such as is produced by halving the number of atoms affected, produces comparatively little change in the pattern of the diffracted rays, that is, the distribution of scattered electrons.

The main features of the pattern are determined by the first few elements, just as, on the older view, the direction of the scattered electron would be mostly determined by the forces exerted on it by the atoms near which it goes and comparatively little by the more distant ones.

The easiest way of looking at the whole thing seems to be to regard the waves as an expression of the laws of motion. The uniform motion of Newton's first law is replaced by a simple plane wave, and so on. On this view, the electron remains the reality, and this seems right, for after all it is the electron as a particle which is actually detected in any practicable experiment, and the waves come to bear the same sort of relation to it that Newton's or Einstein's law of gravitation bears to the planets which obey it.

It is remarkable how near this view comes to that proposed by Newton in the case of light, which as we have seen is analogous, and I can scarcely give a better statement of the position, allowing for slight changes in terminology, than by quoting what he says:

"Those that are averse from assenting to any new discoveries but such as they can explain by an hypothesis, may for the present suppose, that as stones by falling upon water put the water into an undulatory motion, and all bodies by percussion excite vibrations in the Air; so the Rays of Light, excite vibrations in the refracting Medium or Substance . . . that the vibrations thus excited are propagated in the refracting or reflecting Medium or Substance, much after the manner that vibrations are propagated in the Air for causing Sound, and move faster than the rays so as to overtake them . . . and that every Ray is successively disposed to be easily reflected or easily transmitted, by every vibration which overtakes it. But whether this Hypothesis be true or false I do not here consider."

In what follows Newton contents himself by describing the experimental facts. But his hypothesis, the guess of genius, after a century of discredit, is reasserting its superiority over the more logical, but less inspired, ideas of the early nineteenth century.

The Initiation of Respiration at Birth.

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IN a classic paper forty years ago, Miescher-Rusch reviewed the evidence then available in regard to the control and regulation of respiration in the adult, and also in regard to the initiation of respiration at birth. He pointed out that afferent nervous stimulation is a merely accessory influence, and that the blood gases are the dominating control with carbon dioxide in the major rôle. He did no experiments, however, to prove this thesis.

About twenty years ago the experimental

demonstration and proof were developed by Hal-dane with Priestley and Douglas that in man, and by inference in all mammals after the independent life of the individual is established, respiration is chiefly controlled by carbon dioxide. But the conditions causing the initiation of breathing at birth have remained until now undemonstrated, and their relations undefined. The belief that it is the chill of exposure to the air that starts the new-born to breathing has come down from a remote past, and is still held unquestioned by