Recent Progress in Theoretical Physics.

By H. F. BIGGS.

WHEN an army is making a rapid advance over fresh ground it is not the time to make an elaborate survey of the territory conquered, but on the other hand sketch maps, however provisional and imperfect, of the main lines of advance and the chief points of attack, are all the more useful to the soldier in the field and interesting to the newspaper reader at home. What follows is an attempt to draw such a sketch map for some important recent work in physics.

The most remarkable thing about present-day physical theories is the strange dual nature all the components of matter and energy seem to exhibit. Thus no sooner does Bose (*Zts. f. Phys.*, **26**, 178; 1924) show that Planck's law of temperature radiation can be derived by treating quanta $h\nu$ as if they were particles with momentum $h\nu/c$, than Einstein (*Berl. Ber.*, 261; 1924) follows suit by applying Bose's method to the molecules of a material gas, supporting his procedure by treating his molecules as equivalent to wave-trains after the fashion of L. de Broglie. A new statistical principle is introduced by Bose, but it will be easier to explain this after we have considered the rival principle commonly called the Fermi-Dirac statistics. This latter principle may be stated in a few words as the extension to quantised translational motion of Pauli's prohibition, by which no two electrons in an atom can have exactly the same set of quantum numbers.

But how are translational motions of particles to be quantised? This question can be most easily answered by considering the de Broglie waves of our particles (L. de Broglie, *Ann. de Phys.*, **3**, 22; 1925; or Biggs, "Wave Mechanics," p. 22). A moving particle is equivalent to a train of waves (ψ -waves) the wave-number of which is p/h , where p is the momentum of the particle and h is Planck's constant. The components, too, of the momentum are proportional to the wave-numbers along the co-ordinate axes, that is, to the number of waves per centimetre cut by the axes respectively. Thus we write

$$N_x = p_x/h, \quad N_y = p_y/h, \quad N_z = p_z/h \quad (1)$$

The elastic rebound of a molecule from a boundary wall is now equivalent to the reflection of ψ -waves from the wall, since the laws for angles are the same, and thus an ideal gas in an enclosure with perfectly elastic walls is replaced by a field of ψ -waves in an enclosure the walls of which are perfectly reflecting for these waves. This only means that if there is to be no progressive motion of particles or radiation through any surface drawn in a gas, the wave-system must be a stationary system such as would be formed by trains of waves reflected to and fro at the surface. The problem of velocity distribution in a gas is then closely related to the Rayleigh-Jeans problem of finding the number of degrees of freedom or modes of oscillation in a given volume of a continuous

medium. (See Jeans, "Dynamical Theory of Gases," chap. xvi.; 1921.)

A less mathematical argument than the usual one may be used, and is more appropriate to the case in hand. Let the oblique lines in the diagram (Fig. 1) be the traces of wave-fronts in the original train and in a train formed by reflection at some vertical wall, the full lines representing crests and the dotted lines troughs. Then the reflecting walls must be either in positions such as w, w , if at nodes of the standing waves, or W, W , if at antinodes. In either case, a line drawn normal to the planes must cut a whole number of half-waves. Thus in a rectangular box, with edges a, b, c , cm. parallel to the axes, $2aN_x, 2bN_y, 2cN_z$ must be whole numbers,

FIG. 1.

where N_x, N_y, N_z are positive quantities giving the wave-numbers without regard to sign. If we then represent the possible wave-trains each by a point the rectangular co-ordinates of which are $2aN_x, 2bN_y, 2cN_z$, we obtain a cubic space-lattice in the positive octant of space along the constant of which is one unit; if, however, we take N_x, N_y, N_z themselves as co-ordinates, the unit cell of our representative space-lattice will be a rectangular parallelepiped of dimensions $1/2a, 1/2b, 1/2c$, and finally by (1), the possible momenta of particles in the box will be represented as to the absolute magnitude of the components by the points of a space-lattice in the positive octant the unit cell of which measures $h/2a$ by $h/2b$ by $h/2c$.

It will be noticed that we have thus quantised the linear momentum of a particle in our box in almost the same way as de Broglie quantised the angular momentum of an electron describing a circular orbit. In our case, it is the number of half-waves along the length, breadth, or depth of the box that must be a whole number; in de Broglie's case, it is the number of waves counted 'round the orbit.' In both cases quantisation expresses the

existence of a steady state, or, from another point of view, the existence of a single-valued function ψ .

We can now see clearly how we are going to get the theory into relation with classical and 'classical-quantum' statistical dynamics. But first we must take a rather different space-lattice for our possible momentum-points, not occupying only the positive octant of space but also extending symmetrically on all sides of the origin. We evidently get the same number of points up to a given maximum value P of the momentum if we take, instead of a spherical octant of radius P , a whole sphere of radius P , provided we multiply the volume of our unit momentum cell by 8, making its edges now $h/a, h/b, h/c$, instead of half these lengths. Then the unit cell in the momentum-space for a particle in a box abc will have the volume h^3/abc , and the unit cell of the 6-dimensional phase-space with co-ordinates x, y, z, p_x, p_y, p_z , will have the volume h^3 . It will not be necessary, however, for our purposes to fuse the real space and the momentum space into a 6-dimensional continuum, but it will be more convenient to keep the conceptions of the two spaces distinct.

Now so far we have postulated no statistical principle whatever, but have left it quite indeterminate *how many particles* there may be to each point of our momentum space-lattice. Leaving until later the Bose-Einstein principle and the classical principle of assigning these numbers, we will follow out some consequences of the answer that arises most naturally from our train of reasoning, namely, that *no two particles can have the same lattice point*, that is, no two particles can have the same momentum in the same direction—and in particular, even at the zero of temperature when the system has radiated all the energy it can radiate, only one particle can have zero momentum or zero energy. This is the Fermi-Dirac principle (Fermi, *Rend. Accad. Lincei*, 3, 145; 1926; Dirac, *Proc. Roy. Soc., A*, 112, 661; 1926). It must hold for the same reason that the similar Pauli principle holds for the electrons in an atom, though what this reason may be is a puzzle. One may perhaps hazard the guess that if two molecules or two electrons had the same wave-trains, or oscillations the same in all but phase, they would no longer be the distinct entities of our experience, the discreteness of motions being thus very intimately bound up with the atomic nature of matter itself.

This reasoning of course is quite right only for *structureless* particles which alone can be fully represented by the oscillation of a scalar such as ψ ; it should hold, therefore, for a monatomic gas for all phenomena that are not influenced by atomic structure, but for the spinning electron there may be *two* electrons instead of one per lattice point, as in the Stoner atomic scheme. The statistical principle is just the same, but the representation loses a little in *Anschaulichkeit*.

For a true particle we may now state the principle thus: *The product of the real space available per particle and the least momentum space which must be assigned to each particle is h^3* . The first consequence of this is that at the zero of temperature the

actual momenta of the particles will not be zero, but their representative points will be close-packed, occupying all those points of our lattice which lie closest to the origin or zero of momentum with one cell of volume h^3/V per particle. Thus, if we have N particles confined within V cubic centimetres, their momentum space will have the volume Nh^3/V ; and since the shape of this volume will presumably be spherical (for N large), we can easily calculate the energy. For if P be the radius of this sphere, we have, to find P

$$(4\pi/3)P^3 = Nh^3/V,$$

and the kinetic energy of a particle with momentum p is $p^2/2m$, so that dividing the total energy

$$\frac{V}{h^3} \int_0^P 4\pi p^2 dp \frac{p^2}{2m}$$

by the number of particles

$$\frac{V}{h^3} \int_0^P 4\pi p^2 dp, \text{ we get for the mean energy}$$

$$\frac{1}{2m} \cdot \frac{3}{5} P^2 \text{ or } \frac{3h^2}{10m} \left(\frac{3N}{4\pi V} \right)^{\frac{2}{3}}.$$

A striking application of this result was made by R. H. Fowler (*Mon. Not. Roy. Astr. Soc.*, 87, 114; 1926; see also Eddington: "Stars and Atoms," Oxford, 1927, Appendix) to the dense matter of the companion of Sirius, showing that even when such matter has radiated all the energy it can, and has cooled down to the absolute zero, the 'free' electrons, of which (as regards number and energy) it is chiefly composed, have a mean energy corresponding to equipartition energies at something like 10^8 degrees.

Some of the mysteries of matter in its more familiar forms can also be illuminated by this principle in the most hopeful way. Thus Pauli (*Zts. f. Phys.*, 41, 81; 1926) shows that the small paramagnetism of metals can be accounted for by treating the free electrons as magnetic molecules of a gas near the absolute zero. For corresponding temperatures are inversely as the masses of the particles concerned, so that to the helium molecule at 5° abs. corresponds electron gas at about $36,000^\circ$. If then we start filling up the lattice points with electrons, two go to each point, and these have opposite orientations, so that the resultant magnetic moment must be zero, to the first approximation. Pauli pursues the analysis further and derives Curie's law ($kT = \text{const.}$) for the susceptibility and finds absolute values which promise to agree well with observation when we know better the intrinsic diamagnetism to be assigned to the various atoms.

More recently, Sommerfeld (*Naturwiss.*, 15, 825; 1927) has attacked other hitherto baffling problems of electronic phenomena in metals with very promising results, building on the Drude treatment by mean free paths, but using the Fermi-Dirac statistics instead of the Maxwell statistics of Lorentz. The great stumbling-block of the negligible contribution of the electrons to the specific heat is surmounted at the first stride, since the degradation or high quantisation of the electronic motions gives a value for the specific heat which

vanishes at 0° abs. (in accordance with Nernst's theorem) and at ordinary temperatures is only of the order $R/100$, which is quite compatible with observation. The formulæ for the electric and the thermal conductivity depend on the mean free path and cannot yet be satisfactorily checked, but in their ratio, the Wiedemann-Franz coefficient, this

unknown divides out, and the result, $\frac{\pi^2}{3} \left(\frac{h}{e}\right)^2 T$,

which is the classical formula with the numerical factor $\pi^2/3$ instead of 2, agrees exactly with the observed mean for the various metals. This, however, is not quite so conclusive as it sounds, since even the seven best metals differ among themselves by 8 per cent. Still more convincing are the results for thermoelectric phenomena, which all come out of the right order, where the classical theory is wrong by very large factors on any reasonable assumptions.

It should be noted that Sommerfeld introduces no arbitrary quantities at all, and the only quantity he uses which varies from metal to metal is the number of electrons per cubic centimetre, which he puts equal to the known number of atoms. One interesting result of this procedure is that the contact potentials come out, for the most part, correct in magnitude but wrong in sign. In other words, an electron would escape from its fellow electrons alone at the given density with a *positive* energy equal to the *work that must be done* to get it out from an actual metal containing the same number of positive atoms with the electrons. Thus, with an obvious notation,

$$W_e = W_a - W_e, \text{ or } W_a = 2W_e.$$

This at once suggests an analogy with the familiar theorem that the potential energy is minus twice the kinetic energy for particles obeying the inverse square law (Sommerfeld, "Atombau," Appendix). For the interactions (if this word can be used at all) of the electrons which cause them to obey this new statistics seem to have nothing to do with their ordinary electrostatic repulsions, so that W_e calculated by the statistics will presumably correspond to kinetic energy, and W_a to potential energy.

There can be no doubt that this paper of Sommerfeld's forms the first¹ step in the true theory of metallic conduction, and that the preponderating factor in these phenomena depends on this statistical behaviour of the electrons without regard even to the structure of the metallic crystals. The next step, as Sommerfeld remarks, will be to improve on the assumption of a constant mean free path and to discover how this quantity depends on the velocity of the electrons and on the temperature.

Let us now return to the question of the different possible statistical principles and trace out the connexion between them. All statistical problems can be regarded as depending on the number of ways a number of objects can be put into a number

of boxes with different labels under various rules. For gases the objects have until now been molecules and the boxes cells of phase space. Classically, the cells are taken equal in size, the intrinsic probability of a molecule being in one of these cells is the same for any cell, and finally the cells are made infinitely small. In the classical quantum theory of Nernst and many other workers, the only difference is that the cells are not made infinitely small but have the finite size h^3 ; they still are regions of equal size and of equal probability. But for both the new statistical principles the point of view is entirely different; we consider, not into which cell we put a particle, but how many particles a cell contains. On the Fermi principle, each cell may contain one particle, but not more, and it is further assumed that to contain one particle and to contain no particle are equally probable states for a given cell. In terms of objects and boxes, it is the cells that are now the objects, and there are only two boxes, which have equal chances of receiving a given object (*i.e.* cell), one labelled 'no particle' and the other 'one particle.'

As we have seen, there seems to be some sort of intelligible reason for this principle, but the Bose-Einstein principle is thoroughly baffling² as to reasons, though easy enough to express. It runs: *It is equally probable that a cell will contain any number of particles.* Or, with the cells still as objects, the boxes of equal probability are labelled *0 particle, 1 particle, 2 particles*, and so on. Jordan (*Naturwiss.*, 15, 636; 1927)³ illustrates the application of the three principles to the simple case of two particles and two cells by this diagram (Fig. 2).

FIG. 2.

In the classical quantum theory we can suppose the particles named *a* and *b*. Then the arrangements with both in the first cell and with both in the second cell will have one complexion each and will be equally probable, while the arrangement with one in each cell will have two complexions and will therefore be twice as probable. On the Bose principle, on the contrary, the particles have lost all individual distinction and can be represented by mere dots. The arrangement with one particle in each cell has then only one complexion and has the same probability as either of the arrangements with both particles in one cell. On the Fermi principle the only possible arrangement is with one particle in each cell. Then, on any theory, once

² See, however, a short and most suggestive paper by Brillouin (*Comptes rendus*, 184, 589; 1927), in which he shows that in the Bose scheme the probability of a quantum getting into a cell is *greater* the more quanta there are already in that cell. It is like the game of chain-tig, in which the more boys there are in the chain the greater is the chance of their adding to their number. This is only a fanciful way of putting the fact that the greater the radiation density corresponding to a certain energy-jump, the greater is the probability of the radiation provoking another such jump, as in Einstein's proof of Planck's law in 1917.

³ In this and a paper in the previous number Jordan gives a most comprehensive review of the progress of the new theories in the last two years, with a full bibliography.

¹ This is not quite fair to Lindemann, who some time ago suggested that the electrons form a nearly rigid space-lattice among themselves (F. A. Lindemann, *Phil. Mag.*, 29, 127; 1915). So they do, but it is in the momentum space, not the real space.

the intrinsic probability is settled, the steady state of maximum probability can be found by the familiar methods.

We have seen that the Fermi principle can be used for electrons not attached to particular atoms, and Fermi in his original paper (*l.c.*) showed that it gave what is probably the true equation of state for a monatomic gas at low temperatures, while Einstein's application of Bose's statistics to gases led to curious results not confirmed by experiment.

It seems, therefore, that Bose's original application of his statistical principle to quanta is the only permissible one. Why quanta indistinguishable in every respect should be capable of coexisting when electrons are not is by no means obvious. Quanta seem to be much more indifferent to each other's existence than material particles, which perhaps has something to do with their own indifference to life, or rather their readiness to undergo metempsychosis.

Obituary.

DR. F. RASCHIG.

DR. FRIEDRICH RASCHIG, whose numerous scientific investigations in the domain of chemical technology covered a remarkably wide range and whose interests were not restricted to purely technical problems but were devoted to public welfare, passed away on Feb. 4 at the comparatively early age of sixty-four years.

We are indebted to the *Chemiker-Zeitung* for some points in Dr. Raschig's life. Born on June 8, 1863, at Brandenburg, Raschig's interest in scientific work was awakened while still at the *Realgymnasium*, where he came under the influence of Prof. Müller, director of the *Zeitschrift für physikalischen und chemischen Unterricht*. After studying at the Universities of Berlin and Heidelberg, he became assistant in the University chemical laboratories in Berlin and was afterwards appointed chemist to the Badische Anilin und Soda Fabrik. During this period many publications, dealing chiefly with compounds of nitrogen and sulphur, appeared under his name. In 1891 he started a factory for the manufacture of phenol, but soon afterwards he resumed his studies of the reactions involved in the lead-chamber process for the production of sulphuric acid. The lengthy controversies aroused by this work no doubt did much to stimulate further research in this important field. He also devoted much attention to the reactions of sulphurous anhydride and the sulphites.

Dr. Raschig discovered a technical method of preparing hydroxylamine, and his investigation of chloramine led to his well-known process for the manufacture of hydrazine, a compound of considerable interest, which had previously been discovered by Curtius, whose death occurred only a few days after that of Raschig. The latter also elaborated many useful volumetric methods of analysis and was an authority on the manufacture of synthetic phenol and the distillation of coal-tar. Dr. Raschig took an active part in municipal and political life, and in 1924 was elected a member of the Reichstag.

THE issue of the *Physikalische Zeitschrift* for Dec. 15 contains a photograph and a short account of the life and work of the late Prof. A. Gockel, of Freiberg, Switzerland. Gockel was born in November 1860 at Stockach in Baden, where his father was a secretary in the Post Office. After attending the Gymnasium at Constance he was in turn a

student at the Universities of Freiburg in Baden, Würzburg, Carlsruhe, and Heidelberg. He graduated in 1885 and held teaching posts in secondary schools for ten years. In 1895 he became assistant to Prof. Kowalsky at the University of Freiberg, Switzerland, and in 1901 was appointed lecturer, two years later extra professor, and in 1910 ordinary professor of cosmical physics. In 1921-22 he acted as Rector of the University. He died on Mar. 4 of last year. He married in 1902 the daughter of his colleague Baumhauer, professor of mineralogy. The whole of his scientific publications deal with atmospheric phenomena such as thunderstorms, radiation, and electromagnetic waves, and his work carried him to the shores of the Mediterranean and into the Sahara. His papers are characterised by the carefulness of their deductions.

WE regret to announce the following deaths:

Prof. W. Steadman Aldis, formerly professor of mathematics and principal of the College of Science, Newcastle-on-Tyne, and later of Auckland College, New Zealand, on Mar. 7, aged eighty-nine years.

Mr. W. B. Croft, who taught mathematics at Winchester College for many years, and was widely known for his interest in early work on radiotelegraphy and in optics, on Mar. 23, aged seventy-six years.

Prof. W. W. H. Gee, formerly professor of pure and applied physics at the College of Technology, Manchester, and author of papers and text-books on physics and electro-chemistry, on Mar. 3, aged seventy years.

Dr. F. S. Luther, emeritus president and formerly professor of mathematics of Trinity College, Connecticut, on Jan. 4, aged seventy-seven years.

Mr. E. W. Maunder, for many years Superintendent of the Solar Department of the Royal Observatory, Greenwich, on Mar. 21, aged seventy-six years.

Dr. Richard Pribram, emeritus professor of chemistry in the University of Prague, whose work on the optical rotation of organic substances and on the relationship between physical properties and chemical constitution is well known, aged eighty years.

Prof. Herbert M. Richards, professor of botany in Barnard College, Columbia University, and scientific director of the New York Botanical Garden, on Jan. 9, aged fifty-six years.

Mr. A. Shoolbred, author of "The Flora of Chepstow" (1920), a well-known local botanist and horticulturist, on Jan. 25, aged seventy-five years.

Prof. Eilhard Wiedemann, well known for his writings on the history of physics and formerly professor at the University of Erlangen, at the age of seventy-six years.