to reduce the equation of a cubic to four simple canonical forms. Each of these was discussed in detail, and its double points, ovals, etc., were considered. He found seventy-two of the possible seventy-eight forms that a cubic may take. Finally, he stated without proof that all cubic curves could be derived by projection from the canonical form

$$y^2 = ax^3 + bx^2 + cx + d,$$

a theorem which remained unproved until 1731. Apart perhaps from some of his geometrical investigations, Newton's pure mathematics was dominated by his invention of the calculus and his efforts to solve the problems of Nature. No one has surpassed him in combining theory and its application. No one appreciated him more than the greatest of the great mathematicians.

Newton's Work in Optics.

By Sir RICHARD GLAZEBROOK, K.C.B., For. Sec. R.S.

NEWTON entered Trinity College, Cambridge, in 1661 shortly before his nineteenth birthday, and proceeded to his B.A. degree in 1665. On Oct. 1, 1667, he became a fellow of the College, and in 1669 was chosen as Lucasian professor of mathematics in succession to Barrow, whose optical lectures he had helped to edit during the previous year. His own interest in optics began at an early date; while still an undergraduate he had written about haloes. In 1666 he "procured a triangular glass prism to try the celebrated phenomena of colours." His first telescope was constructed in 1668 and his second telescope sent to the Royal Society in 1671. The following year (Jan. 11, 1671, old style) he was elected a fellow of the Royal Society, and his letter containing his new theory of light and colours was read on Feb. 6, 1672. Hooke, who with Boyle and the Bishop of Salisbury was asked to report on the paper, wrote a critical report, claiming that Newton's results were contained in his own "Micrographia" published in 1664. Criticisms were also published by Linus of Liège and Lucas.

Newton was admitted as a fellow of the Royal Society in 1675, and shortly afterwards sent to the Society a paper on the colours of thin plates. Hooke was again a critic. The main part of the work, he said, "was contained in the 'Micrographia,' which Mr. Newton had only carried further in some particulars."

Some correspondence followed, and Newton in his final letter gave Hooke full credit for his work. "You," he wrote, "have added much several ways, and specially in considering the colours of thin plates. If I have seen farther it is by standing on the shoulders of giants."

It was not until some thirty years later (1704) that the first edition of the "Optics" appeared. A second edition with additions—"A Treatise," it is called, "of the Reflexions, Refractions, Inflexions, and Colours of Light"—was printed by the printers to the Royal Society in 1718, fifty years after the construction of his first telescope, and contains, in

a series of Queries of very real interest, his latest views on optics and allied subjects.

But before dealing with the questions raised in these, some reference to the book itself is called for. The difficulties which followed the reading of his papers in 1675 were the main cause of the delay in publication. "To avoid being engaged in Disputes about these Matters," he writes, "I have hitherto delayed the printing, and should still have delayed it had not the Importunity of Friends prevailed upon me." The treatise is divided into three books. The first deals with the reflexion, refraction, and dispersion of light; the second with the colours of thin and thick plates; and the third with diffraction, and in the Queries with double refraction and the theory of light.

The first book opens with a statement of known propositions as a series of definitions and axioms. "For what hath hitherto been generally agreed on I content myself to assume under the Notion of Principles. . . . And this may suffice for an Introduction to Readers of quick Wit and good Understanding not yet versed in Optics, although those who are already acquainted with this Science and have handled Glasses will more readily apprehend what followeth," and then we come to Prop. I. Theor. I. "Lights which differ in Colour differ also in Degrees of Refrangibility." From this we pass by a series of experiments most beautifully designed, most clearly described, to an investigation of the chromatic aberration of lenses with an account of the defects of refracting telescopes, incurable, as Newton thought he had proved it to be by these experiments, and the construction of his own reflecting instrument. "Seeing, therefore," he writes, "that the Improvement of Telescopes of given lengths by Refractions is desperate, I contrived therefore a Perspective by Reflexion, using instead of an Object-glass a concave Metal." The mirrors of the first two telescopes he made and polished himself.

"By many trials I learnt the way of polishing till I made those two reflecting Perspectives. . . .

For this Art of polishing will be better learned by repeated Practice than by my Description."

"When I made these an Artist in London undertook to imitate it, but using another way of polishing them than I did he fell much short of what I had attained to, as I afterwards understood by discoursing the under Workman he had employed."

Newton was mistaken, as we now know. Fig. 9—Fig. 26, Plate V., of the first book of the "Optics"—illustrates the crucial experiment (Exp. 15, Bk. I. Part I.). In the figure PT is the position of the spectrum formed on a vertical screen by a prism with its edge horizontal and its angle downwards. The plane of the screen is parallel to the edge of

FIG. 9.—Reproduced from Newton's "Optics."

the prism. The light forming this spectrum is intercepted by a second prism placed close behind the first, but with its edge vertical; the spectrum is displaced into the position *pt*. *Tt* and *Pp* measure the tangents of the deviation of the violet and red ends respectively, while their difference depends on the dispersion. *2t2p*, and *3t3p* represent spectra formed similarly by other prisms placed in turn in the position of the first vertical prisms. *S* is the position of the spot formed on the screen by the direct light of the sun, painting "his round white image when the prisms were taken away."

Newton observed that when the angle of the prism forming the image *3p3t* was equal to that of the first prism, the axis of the spectrum *3t3p* met that of the original spectrum *TP* at the centre of the sun's image *S*, while the produced axes of the spectra *tp* or *2t2p* formed by prisms of less angle cut the axis of *TP* in the points *m*, *n* a little

beyond the centre of the image *S*. From the figure he obtained the values of the tangents of the deviations of the violet and red rays in each case

"from whence the Proportions of the Sines being derived, they come out equal so far as by viewing the Spectrums and using some mathematical Reasoning I could estimate";

or, in other words, if we denote by α , β , the deviations of two rays (violet and red, say) produced by a prism, then the ratio $\sin \alpha / \sin \beta$ is independent of the prism.

It is, however, not clear from the description that the prisms were of different material. With modern glasses of high or moderate dispersion the axes of the spectra would not be straight, but curved, and would not appear to meet near *S*.

Newton refers to the different angles of the several prisms, and continues:

"But for want of solid Glass Prisms with Angles of convenient bignesses, there may be Vessels made of polished plates of Glass cemented together in the form of Prisms and filled with Water."

It is at any rate possible, or perhaps rather probable, that he proved the proposition only in the case of prisms of the same material, and in any case it is not likely that he used prisms of widely different optical properties, even though they may not all have been bought at Stourbridge Fair, to try the celebrated phenomena of colours. And so he concludes:

"But by reason of this different Refrangibility I do not see any other means of improving Telescopes by Refractions alone than that of increasing their lengths, for which end the late Contrivance of Hugenius seems well accommodated."

But to pass on; other propositions of the book deal with the nature of white light and the composition of colours.

"Colours," it is shown, "may be produced by Composition which shall be like to the Colours of Homogenous Light as to the appearance of Colour, but not as to Immutability of Colour and Constitution of Light";

and again:

"Whiteness, and all grey Colours between white and black, may be compounded of Colours, and the whiteness of the Sun's Light is compounded of all the primary Colours mixed in a due Proportion."

In another proposition it is shown that if light be due to the motion of corpuscles moving in the direction of the ray, but subject, when in the neighbourhood of the surface of separation of two different media, to a force normal to the surface, and if v , v' be the velocities in the two media, ϕ , ϕ' the angles

of incidence and refraction, then $\sin \phi / \sin \phi' = v'/v$. Thus if ϕ is $> \phi'$, then v' is $> v$, or, of the two media, the velocity is greater in the more dense, contrary to what we now know to be true.

So far Newton has been dealing with geometrical optics. Book II. introduces us to "Observations concerning the Reflexions, Refractions, and Colours of thin transparent Bodies"—to Newton's rings and the theory of light he devised to account for his observations.

Hooke in 1665, and Boyle two years earlier, had observed the colours shown by thin sheets of mica in both transmitted and reflected light, and Hooke ("Micrographia," p. 47) had described how to produce these colours by placing two lenses in contact and reflecting light from the thin film of air between them. Newton, after a brief reference to the observations of others, describes his own experiments. He found the squares of the radii of the bright rings to be in the arithmetical progression of the odd numbers 1, 3, 5, . . . and those of the dark rings in the arithmetical progression of the even numbers 0, 2, 4, 6, . . .

A knowledge of the radii of curvature of the lenses and of the radii of the rings enabled him to calculate the thicknesses of the air film for light and dark rings. These he found to be, in inches, $1/178000$, $3/178000$, . . . and $0, 2/178000, 4/178000$. . . respectively.

Having now obtained his facts, Newton attempts to explain them:

"Every Ray of Light," he writes (Bk. II. Pt. iii. Prop. xii.), "in its passage through any refracting Surface is put into a certain transient Constitution or State which in the progress of the Ray returns at equal Intervals and disposes the Ray at every return to be easily transmitted through the next refracting Surface, and between the returns to be easily reflected by it." And then follows a definition: "The returns of the disposition of any Ray to be reflected I will call its Fits of easy Reflexion, and those of its disposition to be transmitted its Fits of easy Transmission, and the space it passes between every return and the next return the Interval of its Fits."

He has no theory to account for this, but writes:

"What kind of action or disposition this is; whether it consists in a circulating or a vibrating Motion of the Ray or of the Medium or something else, I do not enquire."

He postulates (Query 17) a refracting or reflecting medium in which waves of vibration or tremors are excited at a point in which a ray of light is incident, "and continue to arise there and be propagated from thence as long as they continue to do so." While (Query 18) he writes, after

describing an experiment which shows that heat can reach a thermometer when suspended in a vacuum:

"Is not the Heat of the warm Room conveyed through the *Vacuum* by the Vibrations of a much subtiler Medium than Air, which after the Air was drawn out remained in the *Vacuum*? And is not this Medium the same with that Medium by which Light is refracted and reflected, and by whose Vibrations Light communicates Heat to Bodies and is put into Fits of easy Reflection and easy Transmission? And do not the Vibrations of this Medium in hot Bodies contribute to the intenseness and duration of their Heat, and do not Hot Bodies communicate their Heat to contiguous cold ones? And is not this Medium exceedingly more rare and subtiler than the Air, and exceedingly more elastick and active? And doth it not readily pervade all Bodies? And is it not (by its elastick force) expanded through all the Heavens?"

And again, Query 19:

"Doth not the Refraction of Light proceed from the different density of this Ethereal Medium in different places, the Light receding always from the denser parts of the Medium? And is not the density thereof greater in free and open Spaces void of Air and other grosser Bodies than within the Pores of Water, Glass, Crystal, Gems, and other compact Bodies?"

These queries appeared in 1718. In the text of the "Optics" (1704), Newton was more cautious, for in Prop. xii., after his inquiry as to the action or disposition which causes the alternate fits of reflexion and transmission, he permits "those who are averse from assenting to any new Discoveries but such as they can explain by an Hypothesis" to "suppose for the present" that "the Rays of Light by impinging on any refracting or reflecting Surface, excite vibrations in the reflecting or refracting Medium or Substance."

We will return below to the question of why this complexity was necessary.—A stream of corpuscles constituting a beam of light, accompanied in their path by waves in an elastic medium travelling with a velocity exceeding that of the corpuscles themselves, overtaking them and jostling them into alternate fits of reflexions and transmission, a theory so complex that in the words of Verdet:

"Pour renverser ce pénible échafaudage d'hypothèses indépendantes les unes et les autres il suffit presque de le regarder en face et de chercher à le comprendre."

Meanwhile some brief account must be given of the Third Book of Optics: "Observations concerning the Inflexions of the Rays of Light and the Colours made thereby." It deals with Grimaldi's observation of the colours of bands

seen on either side of the shadow of a hair illuminated by light from a small hole—the diffraction of light.

Newton experimented with sunlight transmitted into his darkened chamber through a pinhole $\frac{1}{42}$ part of an inch in diameter. "For 21 of those Pins laid together took up the breadth of half an inch"; he measured the breadth of the fringes, three in number, at various distances from the hair, and extended the observations to include the fringes produced by a sharp knife edge and also by a narrow triangular slit formed by two knife edges inclined at a small angle to each other. Of these appearances he attempts no explanation, stating that at the time he was interrupted, "and cannot now (1718) think of taking these things into consideration. And since I have not finished this part of my Design, I shall conclude with proposing only some Queries in order to a further search to be made by others," and the first queries ask:

"Do not Bodies act upon Light at a distance and by their action bend its Rays, and is not this action (*cæteris paribus*) strongest at the least distance? Do not Rays which differ in Refrangibility differ also in Flexibility? Are not Rays of Light passing by the edges and sides of Bodies bent several times backwards and forwards; with a motion like that of an Eel? And do not the three fringes of coloured Light above mentioned arise from three such bendings?"

In Query 25 we come to the double "refraction

of Island Crystal described first by (1670) Erasmus Bartoline and afterwards more exactly by Hugenius in his book 'de la Lumière' (1690)." The properties of the crystal and its effects on light are described, ending with the Query (26):

"Have not the Rays of Light several sides endued with several original Properties?"

This is discussed and proved, leading to a further Query (28):

"Are not all Hypotheses erroneous in which Light is supposed to consist in Pression or Motion propagated through a fluid Medium? . . . for Pression or Motion cannot be propagated in a Fluid in right Lines beyond an Obstacle which stops part of the Motion, but will bend and spread every way into the quiescent Medium, which lies beyond the obstacle?"

As a further objection he writes, after a reference to Huyghens' experiments on polarisation:

"For Pressions or Motions, propagated from a shining Body through an uniform Medium, must be on all sides alike; whereas by those Experiments it appears that the

Rays of Light have different Properties in their different Sides."

Thus Newton is led to the conclusion expressed in Query 29:

"Are not the Rays of Light very small Bodies emitted from shining Substances? For such Bodies will pass through uniform Mediums in right Lines without bending into the Shadow which is the Nature of the Rays of Light."

In 1675 he had been less positive; in his series of

FIG. 10.—Statue by Roubiliac in the ante-Chapel of Trinity College, Cambridge, given to the College by Dr. Robert Smith, Master, 1742–68.

"Hypotheses as to the Nature of Light," submitted in that year to the Royal Society, he wrote, after premising the existence of an ether in which waves were set up by the impact of light rays on a reflecting or refracting surface :

"In the fourth place therefore I suppose light is neither æther nor its vibrating movement, but something of a different kind propagated from lucid bodies. To avoid dispute and make this hypothesis general let every man take his fancy. Fifthly, it is to be supposed that light and æther mutually act upon one another."

Newton was aware of Huyghens' "*Traité de la Lumière*," with its beautiful construction for determining the position at any instant of a wave front, in a medium either crystalline or isotropic, as the envelope of the wavelets proceeding from all points of a previous position of the front; but such a construction did not account for rectilinear propagation. He was aware, also, of the fact that two waves in water of the same period reaching a given point by paths differing by an odd multiple of half a wave produced rest, and had thus explained the tidal phenomena of the port of Batcha in Cochin China, an explanation which one hundred years later first put Young on the track of interference; but he failed to see how this would account for many of the experimental results he had so clearly described.

Newton knew, also, that sound waves in air were propagated by disturbances taking place in the direction in which the waves are travelling, and, for him and his contemporaries, the waves in the luminiferous medium possessed this same property. Such a motion could not account for polarisation, and though he refers several times in the "*Optics*" to the vibrations of a stretched string, no one in his day had conceived that the vibrations to which light is due could, like those of the string, be transverse and give to the ray the sides needed to account for polarisation.

A century passed before Newton's doubts were resolved, and for that period his views held the field until, on Nov. 12, 1801, Young applied the principle of interference to explain the diffraction of light, the colours of thin plates, and many of the other phenomena discussed by Newton. Fresnel's first memoir on diffraction is dated 1815, and was followed in rapid succession by other researches by which he established securely the consequences of interference, and showed how it, when combined with the extreme smallness of the wave-length of light, led to an explanation of

the rectilinear propagation. His memoir on the diffraction of light was crowned by the French Academy in 1818, exactly a century after the publication of the second edition of the "*Optics*."

Newton's second difficulty was solved at the same time. Hooke had suggested, but without any convincing reasons, that the vibrations in a light ray were transverse to the ray. Fresnel in 1816, in a Memoir on the influence of polarisation on the mutual action between two rays of light, makes the same hypothesis. Young in 1817 (article '*Chromatics*,' Supplement to "*Encyclopædia Britannica*") writes :

"The elementary motions of the particles of the luminiferous medium are supposed to be principally confined to the line of direction of the undulation, while the most sensible effects of the waves depend immediately on their ascent and descent in a direction perpendicular to that of their progressive motion. . . . If we assume as a mathematical postulate in the undulatory theory that a transverse motion may be propagated in a straight line, we may devise from this assumption a tolerable illustration of the subdivision of polarised light by reflexion on an oblique plane."

But, writes Verdet, "The idea of transversal vibrations appeared a mechanical absurdity to all contemporary men of science," especially to Laplace and to Arago. The latter could never in his life decide to admit it as true, and Fresnel abandoned for a time an explanation based on this hypothesis.

In 1819, Arago and Fresnel published a Memoir on the mutual action between two rays of polarised light, and as a result of this, in 1821, Fresnel adopted the hypothesis of transversality in his *Memoirs on mechanical considerations on the polarisation of light and on double refraction*. One other experiment was required to disprove the emission theory, and this was first carried out by Foucault, who showed that light travelled more slowly in water than in air in accordance with the requirements of the undulatory theory. Newton's theory, as we have already seen, required the reverse result.

Another century has passed, and we have the quantum theory of radiation enunciated by Planck in his famous paper of 1901. Radiant energy leaves a radiant body, not continuously but in definite 'chunks'—quanta—depending on the frequency and on a constant, Planck's constant h , of which the value is 6.55×10^{-27} erg sec. The energy of the system is not equally distributed among the various modes of vibration, as required by classical mechanics; but little is found in short-

period waves; it is mostly concentrated in the low-frequency vibrations.

The law of radiation developed by Planck has been fully verified by numerous experiments, and the inverse problem, given that the final partition of energy is that determined by Planck's law, what laws of motion must be postulated for the system in order to obtain this law, was first solved by Poincaré (1912), who wrote: "L'hypothèse des quanta est la seule qui conduise à la loi de Planck."

Light is one form of radiant energy. Light then leaves a luminant body in quanta—light quanta—and this view has been developed by Einstein in his papers on light quanta (1905) and the theory of radiation (1917). Such a theory has its difficulties; it contradicts at first sight much that we know from experiment about the interference of light. Thus Prof. H. A. Lorentz (quoted by Jeans in his "Report¹ on Radiation and the Quantum Theory") said at the Birmingham meeting of the British Association in 1913: "Now it must, I think, be taken for granted that the quanta can have no individual and permanent existence in the ether, but they cannot be regarded as accumulations of energy in certain minute spaces flying about with the speed of light. This would be in contra-

diction with many well-known phenomena of interference and diffraction. It is clear that if a beam of light consisted of separate quanta, which of course ought to be considered as mutually independent and unconnected, the bright and dark fringes to which it gives rise could never be sharper than those that would be produced by a single quantum."

While it is tempting to try to draw an analogy between Newton's corpuscles and light quanta, and possibly between his fits of easy reflexion and transmission and the probabilities of periodic changes in the states of motion of an atomic system from which the emission of quanta arise, there are difficulties in the way of the acceptance of any mechanical theory purporting to account for the emission of light, which, up to the present, have proved insuperable, unless, indeed, we may see in the new mechanics developed by Heisenberg and Schrödinger some means of bringing the various experimental results under some general law.

In a recent paper read before the Royal Society on the quantum theory of the emission and absorption of radiation, Dirac studies the interaction of an assembly of light quanta with an ordinary atom, and claims to show that it gives Einstein's laws for the emission and absorption of radiation.

¹ For a discussion of the quantum theory, and of course of the problems here alluded to, reference should be made to this Report.