may be a possible explanation of the new stars which have appeared from time to time.

There are two other contributions of Newton to astronomy to which I should like to make a brief reference. In the "System of the World," a work which Newton intended at one time to form Book III. of the "Principia," there is a little-known estimate of the distance of the fixed stars. As it was one hundred years before any better estimate was obtained, I venture to direct attention to it.² The disc of Saturn, Newton says, is 17" or 18" in diameter. It receives $1/2,100,000,000$ th of the sun's light. If we suppose Saturn to reflect $\frac{1}{4}$ of the light it receives, the whole light reflected from its illuminated hemisphere will be $1/4,200,000,000$ th of the light emitted from the sun's hemisphere. If the sun were $10,000\sqrt{42}$ times more distant than Saturn, it would yet appear as lucid as Saturn now does without its ring. Let us, therefore, suppose that the distance from which the sun would shine as a fixed star to exceed that of Saturn by about 100,000 times. This gives 0".2 as the annual parallax of a first magnitude star and 0".002 as its diameter. Newton answers the objection that light may be absorbed in its passage through space, for in that case the remoter stars would scarcely be seen.

² Schlesinger, *Publ. Astr. Soc. Pacific*, vol. 14, p. 170.

We may also recall the fact that Newton was the first person who actually constructed a reflecting telescope. He gives an account in the *Phil. Trans. Roy. Soc.* of the chemical experiments he made for the best composition of speculum metal. The use of pads of pitch on which rouge is spread for polishing mirrors and lenses was invented by him. I believe instrument-makers have found no better materials since.

Returning to Book III. of the "Principia," nowhere has so much been comprehended in so few words as in the law of gravitation. It does not explain to us why the moon and planets move as they do, but it does show us how they move and enable us to calculate their movements. Generations of mathematicians and astronomers have employed large parts of their lives in tracing its consequences in most intricate details, comparing the results with observations, and discerning the past and future history of the solar system. Laplace, perhaps the most illustrious of Newton's followers, says "the number and generality of Newton's discoveries relative to the system of the world, the multitude of original and profound views, which have been the germ of the most brilliant theories of the geometers of the last century, will assure to the 'Principia' a pre-eminence above all other productions of the human intellect."

Newton's Work in Mechanics.¹

By Prof. HORACE LAMB, F.R.S.

IF we wish to realise the extent and importance of Newton's contributions to the science of Dynamics, the most obvious way is to compare the long series of propositions contained in the first two books of the "Principia" with the brief record of what had been accomplished by his predecessors and contemporaries. To both he makes ample acknowledgment. Galileo, freeing himself from scholastic tradition, and appealing to experiment as the ultimate test, had ascertained the laws of falling bodies and, assuming the composition of motions, had deduced the parabolic path of a projectile. Considering, further, motion on a smooth incline, he had proved that the velocity acquired from rest must depend only on the vertical height fallen through, and had supported this theoretical result by an ingenious experiment. It is remarkable that this particular result should have been adopted by Huyghens as the basis of his treatment of the cycloidal pendulum instead of the more natural method of

resolution of forces, which we should expect. Huyghens, for some reason, seems not to have been satisfied with Galileo's demonstration, and argued that if the theorem did not hold it would be possible for a particle descending one incline and ascending another to pass from rest to rest at a higher level. We have here, of course, a partial apprehension of the principle of energy. His work on the compound pendulum, the first and for a long time the only example of what we call Rigid Dynamics, was based on a similar argument, the assumption here being that a system of particles cannot of itself move from rest in such a way as to raise its centre of gravity. Huyghens, again, had published (without proof) his theorems on centrifugal force, and simultaneously with Wren and Wallis had investigated the laws of collision of bodies. He has the distinction of being mentioned by Newton more than once, with especial respect.

This brief outline embraces, I think, almost all that is important from the point of view of pure Dynamics. The advance in knowledge within a period of little more than half a century was, it is

¹ Address delivered on Mar. 19 at Grantham in connexion with the commemoration of the two-hundredth anniversary of the death of Sir Isaac Newton.

true, remarkable, and still more the changed spirit of investigation, with the ultimate appeal to experiment rather than to *a priori* reasoning. Yet although the laws of motion laid down by Newton were, as he asserts, accepted, particular problems were treated separately, each in a special way. There was no systematic procedure based on general principles by which any given dynamical problem could be reduced to a question of mathematics. For example, no case of a *variable* force was attempted, and indeed the whole theory of curvilinear motion, if we except the parabola of Galileo and the cycloid of Huyghens, was unexplored. The mathematical methods also were rudimentary.

The geometrical reasoning of Huyghens, for example, though most ingenious, is special to his problem, and laborious.

When we turn to the "Principia," we find a long array of consequences deduced from the fundamental laws, and by appropriate methods, especially by the consideration of the effect of force on the curvature of an orbit. If we omit from the first eleven sections of the first book a few propositions of mainly geometric interest, and those which are technically astronomical, we find in substance the whole structure of Particle Dynamics much as it is set forth in modern text-books, except for the mathematical methods employed. If we were to reverse what is supposed to have been Newton's procedure and reconvert his geometry into analysis, the result would be a perfectly adequate treatise. This is, in fact, very much what has happened; successive writers have introduced modifications and developments here and there, but the essentials remain the same.

The leading motive which gives a sort of unity to the first book is the law of gravitation with its simpler consequences. We find accordingly the

theory of motion in a conic about a focus, the determination of orbits under given conditions, and even a graphical construction for the time in the orbit. The elliptic problem had been put to Newton by Halley, but no one apparently had even thought of the parabolic and hyperbolic cases, though these were to have an astronomical importance. The first step towards a theory of perturbations is made in the theory of revolving orbits, and the calculation of the motion of the apse when the orbit is nearly circular. Next, abandoning the notion of a fixed centre, Newton solves the problem of two bodies subject to their mutual gravitation, with the correction to Kepler's

third law.

This chain of deductions is brought to a culmination in his celebrated proposition 66, with its long series of corollaries, where the problem of three bodies is attacked in the special form in which it usually presents itself in astronomy. The notion of 'disturbing force,' so fundamental, for example, in the

theory of the tides as well as in gravitational astronomy generally, here appears for the first time.

Although the main theme, as has been said, is the law of gravitation, Newton's instinct for generalisation and for mathematical elegance leads him to discuss various collateral questions which incidentally come within the scope of his methods. For example, the whole theory of central orbits under various laws of force is discussed and practically exhausted, except for the interesting case of the inverse-cube, which was left to be completed by Cotes. The law which comes next in importance to the inverse square, namely, that of the direct distance, with the theory of harmonic motion, elliptic and rectilinear, is fully treated. A number of propositions are devoted to various cases of constrained motion, and we meet here

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FIG. 6.—Manor House, Woolsthorpe, the birthplace of Newton.

with the elegant theorem of the isochronous property of oscillations on an epi- or hypo-cycloid under a central force of this kind. It is, I think, characteristic of Newton, wishing to do things in his own way, that Huyghens' famous result for the ordinary cycloid should be introduced merely as a corollary to this. The first book closes with the theory of attractions of bodies of finite size, including the classical cases of the sphere and spherical shell. Some final propositions on the attractions of stratified media are introduced for their bearing on the corpuscular theory of light.

The second book of the "Principia" deals with a number of questions belonging to the mechanics of fluids, and specially with the theory of resistance. After treating a few standard problems, such as we find in our text-books, on motion under resistances proportional to various functions of the velocity, Newton examines the practical question of how the resistance of the air can be allowed for in pendulum experiments. He has in view here the experimental verification, with as great accuracy as was possible, of his principle that the gravity of a body at a particular place is proportional to its mass. He proceeds to speculate on the nature of the resistance which a fluid opposes to the motion of a body through it. Since this must be due to the momentum communicated to the medium, the law of the square of the velocity is inferred, more or less approximately. Neglecting viscosity, he treats rigorously the case of a medium composed of discrete and inert particles. When he proceeds to continuous fluids such as water, the reasoning is confessedly of a more general character. Incidentally, there occurs a remarkable application of the principle of dynamical similarity, a long anticipation of Fourier.

The remaining propositions deal with hydrostatics and wave-motion. Under the former head we find the law of pressure in an isothermal atmosphere surrounding a globe. Another result, remarkable for the originality of the argument employed, is that if Boyle's law were a statical consequence of repulsive forces between the particles of air, the law of force must be that of the inverse distance. The propositions on waves no doubt owe their insertion to the difficulty which Newton felt in accepting the undulatory theory of light. An ingenious if imperfect attempt is made on the theory of deep-water waves. This is followed by the well-known demonstration of the possibility of sound-waves of harmonic type, and the calculation of their speed of propagation, on the basis of Boyle's law.

From the physical speculations which had attracted some of his contemporaries, Newton carefully refrains. He insists repeatedly that when he refers to accelerating forces or moving forces or centripetal forces he is speaking mathematically, and puts forward no hypothesis as to their ultimate origin. "*Nam virium causas et sedes physicas iam non expendo.*"

This rapid survey necessarily omits much that is interesting. But it should at any rate indicate the enormous debt which the science of Mechanics owes to Newton, for the originality of his conceptions, and for the resolute way in which problem after problem is attacked and made to yield results of interest and beauty as well as of far-reaching importance. It is true, as he indicates, that the fundamental principles had already come to be recognised in a measure, and applied to a few isolated problems. They were now finally vindicated by the host of new and unquestioned results which were shown to flow from them.

Two of the fundamental conceptions were especially due to Newton himself. One is the distinction between gravity and inertia. The other is that of the universal validity of the law of action and reaction, with, in particular, the deduction that the total momentum of a system is unaffected by internal forces. Some appreciation of this is to be found in the work of Huyghens and others on collision, but the generalisation belongs to Newton. A wide extension of the principle is given near the end of the long scholium which he devotes to the explanation and illustration of his third law. It is to be observed that apart from this extension the Newtonian scheme is strictly one of Particle Dynamics, and does not furnish the means of studying in detail the relative motions of the different parts of a solid or fluid of finite dimensions. For this, some additional principle or hypothesis is required, such as was introduced at a later date by d'Alembert and others. It was long overlooked that a sufficient assumption for the purpose is given at the end of the aforesaid scholium. Not only do we find here what is possibly the first general enunciation of the principle of virtual velocities, as distinguished from the partial exemplifications by Galileo and others, but also by the explicit inclusion of accelerating forces the principle becomes identical, when translated into analytical language, with the "variational equation" which Lagrange long afterwards adopted as the basis of his exposition of Dynamics. With this supreme example of the insight of the master this review may fitly close.