it would never convene into one mass ; but some of it would convene into one mass and some into another, so as to make an infinite number of great masses, scattered great distances from one to another throughout all that infinite space. And thus might the sun and fixed stars be formed, supposing the matter were of a lucid nature." I

doubt if modern cosmogony can give a much better account of the genesis of the stars.

So, although two centuries have elapsed since Newton died, we may expect to hear this morning not only of work accomplished more than two centuries ago, but also of scientific methods and lines of thought that seem entirely alive to us to-day.

Newton's Work in Astronomy.¹

By Sir FRANK DYSON, K.B.E., F.R.S.

WRITING in the year 1738, Voltaire narrates that Madame Conduitt, Newton's niece, had related to him that in the year 1666, Newton, seeing an apple fall in the garden of Woolsthorpe, meditated profoundly on the cause which draws all bodies in a line which, if produced, would pass through the centre of the earth. Voltaire then tries to reconstruct the line of thought which led Newton to the law of universal gravitation. Would it act if the apple were a mile high or ten miles high ? If so, might it not act so far away as the moon, and this power, whatever it is, be the same as that which keeps the moon in its orbit round the earth, the planets moving round the sun, and its satellites round Jupiter ?

Vague ideas were current as to the forces which kept the planets in their orbits. Descartes put forward a theory of vortices in a fluid medium diffused through space. This gave a possible sort of mechanism suggested by water or air whirling on the earth, but the details could not be fitted to agree with the known facts of astronomy. Kepler had suggested an attraction between the planets varying as the inverse square of the distance and had afterwards substituted the inverse distance. Others before Newton may have suggested the inverse square, but were unable to carry the idea further. Newton was able to bring the hypothesis to the stage at which it could be tested by its agreement with phenomena. Assuming that the force by which the earth attracted outside bodies varied inversely as the square of the distance and at the earth's surface caused a body to fall 16 feet in one second, he calculated that, as the moon is sixty times as distant from the centre of the earth, the attractive force of the earth at that distance would pull the moon inwards towards the earth 16 feet in one minute. He then calculated the actual force necessary to keep the moon moving round the earth in 27 days 7 hours at a distance of sixty times the radius of the earth. It is said,

but without much evidence, that Newton used the erroneous value of 60 miles for 1° of latitude of the earth's surface employed by mariners to calculate the earth's radius, and was unacquainted with the more accurate measures of Norwood in 1636, which made 1° of latitude 70 miles. He thus obtained a value one-seventh part too small, and in view of this discrepancy for a time turned his thoughts to other things. He may have returned to the subject a few years later when he became acquainted with the results of Picart's triangulation from Paris to Amiens made in 1669-1670.

Though we do not know the precise date when Newton had satisfied himself of the truth of the law of gravitation, in 1684 it was complete, and he could not only demonstrate to Halley that under a law of the inverse square planets would describe ellipses round the sun, but could also promise to send to the Royal Society the mathematical principles of natural philosophy in which the doctrine of universal gravitation is set forth in order and its manifold consequences deduced.

In Book III. of the "Principia" Newton begins with the astronomical evidence. The four satellites of Jupiter describe equal areas about Jupiter in equal times ; and the squares of their periodic times are proportional to the cubes of their distances from Jupiter's centre. The same is true of the satellites of Saturn. The same holds for the motion of the planets round the sun. The moon, too, if the small disturbances caused by the sun are neglected, describes equal areas in equal times about the earth's centre. It follows from the mathematical demonstration in Book I. that the forces which keep Jupiter's satellites in their orbit tend to Jupiter's centre and are inversely as the square of the distances of the satellites from that centre. The same applies to Saturn and its satellites. The movements of the planets round the sun are known to obey these laws of Kepler's with very great accuracy. Consequently this law of the inverse square holds with equal accuracy. The moon, however, does not move accurately in an ellipse,

¹ Address delivered on Mar. 19 at Grantham in connexion with the commemoration of the two-hundredth anniversary of the death of Sir Isaac Newton.

as its apse shifts 3° in each revolution, but this is to be explained by the disturbing action of the sun; but making allowance for this, the force attracting the moon to the earth varies according to the same law of the inverse square of the distance. With the accurate distance of the moon from the earth's centre, the distance it is drawn towards the earth in one minute is calculated, and thus the amount of the earth's attraction at the moon's distance is found. This will be increased 60^2 times at the earth's surface. It is then found to agree exactly with the force which causes bodies to fall at the earth's surface. Thus the force which retains the moon in its orbit is at the earth's surface identical with the well-known property of weight or gravity.

It is, then, gravity which keeps the planets and satellites in their orbits, and all gravitate mutually towards one another, with forces which vary inversely as the square of the distance between their

centres. Newton now investigates whether he can detect any differences in the gravity of different substances. He provided two wooden boxes round and equal; filled one with wood, and suspended an equal weight of gold in the centre of oscillation of the other. The boxes, hanging by equal threads of 11 feet, made a couple of pendulums perfectly equal in weight and figure and equally receiving the resistance of the air. In this way he found that the force of gravity is within $1/1000$ th part the same for gold, silver, lead, glass, sand, common salt, wood, water, and wheat. That is, if the resistance of the air were absent, all those bodies would fall at the same rate.

Newton shows point by point that gravity is a universal property of all bodies, dependent solely

on the amount of matter they contain. This must hold for the planets, and every part of each planet must gravitate to every part of any other planet, and finally every particle of matter attracts every other particle with a force proportional to the product of their masses and inversely proportional to the square of the distance between them. He then shows that two uniform spheres will attract one another as if their masses were concentrated at their centres. This is perhaps the most splendid piece of geometry ever achieved. It appears to have cost Newton a great deal of thought. He says:

FIG. 5.—Portrait by John Vanderbank in the National Portrait Gallery, London.

"After I had found that the force of Gravity towards a whole planet did arise from and was compounded of the forces of gravity towards all its parts, and towards every one part was in the reciprocal proportion of the squares of the distances from the part, I was yet in doubt whether that reciprocal duplicate proportion did accurately hold or but nearly so, in the total force compounded of so

many partial ones; for it might be that the proportion which accurately enough took place in greater distances should be wide of the truth near the surface of the planet, when the distances of the particles are unequal and their situation dissimilar. But by the help of prop. 75 and 76, Book I., and their corollaries, I was at last satisfied of the truth of the proposition as it now lies before us."

The results which follow from the law of gravitation set down in its generality are almost magical. The laws of the movements of the planets round the sun and satellites round the planets follow immediately. Comparison of the dimensions and rate of description of the orbits give the comparative masses of the sun, earth, Jupiter, and Saturn. The sun is 1060 times as massive as Jupiter and 270,000 times (with the parallax Newton used) as

massive as the earth. Thus the earth is found to have a mean density of four times that of the sun, and Newton hazards the guess that the earth's density is between five and six times that of water. The effect of the mutual action of the planets on one another is indicated, and the disturbing action of the sun on the moon is shown to give rise to the revolution of the apse and node, the variation and other inequalities of the moon's orbit. The inequalities known to astronomers were explained and new ones predicted. This complicated problem was carried so far as possible by geometrical methods. Newton said he should give up thinking about the moon, for it made his head ache.

The figure of the earth is next shown to be a spheroid. Owing to rotation the equatorial diameters of planets must be greater than their polar diameters. Observation showed that this was true in the case of Jupiter, which revolves very rapidly in 9 hours 50 minutes. By considering the equilibrium of water in two channels from the pole and a point on the equator to the centre of the earth, Newton showed that the equatorial diameter of the earth would be $1/230$ th part greater than the polar diameter. Assuming a spheroidal form for the earth, he calculated the length of a degree in different latitudes, and the length of a pendulum beating seconds. He compared these with observations made in various places, such as Paris, Cayenne, St. Helena. This investigation served as a great stimulus and provided a theoretical basis for the more accurate determination of the size and figure of the earth.

The general explanation of the tides was a further deduction from the law of gravitation. Newton showed that the attraction of the moon gave rise to forces tending to heap up the water at a point of the ocean directly under the moon and also at the antipodal point; similarly the sun's attraction tended to cause high water directly under the sun and at the antipodal point. He showed how the spring and neap tides would occur, how the position of the sun or moon, north or south of the equator, would affect the tides and how they would vary in different latitudes. By comparing the heights of the spring and neap tides he concluded that the tide-raising force of the moon was 4.48 times that of the sun. Knowing the mass of the sun he finds it should raise a tide of 1 ft. 11 in. in the open sea, and infers for the moon one of 8 ft. 8 in. Newton gave only an elementary theory of the tides, indicating the causes which produced their main features, and left to his successors the difficult problem of a dynamical theory.

Just as the moon produces tides on the earth, so the earth would produce tides on the moon. As the period of revolution of the moon is the same as its period of rotation on its axis, the moon always presents the same face to us. If the moon were covered with water, the water would be heaped up at the part facing the earth and the part diametrically opposite. So Newton infers that the moon has a spheroidal figure, the longer axis pointing to the earth, and being 186 feet in excess of the equatorial diameter.

The precession of the equinoxes is explained as due to the attraction of the sun and moon on the spheroidal part of the earth. Newton had shown how the attraction of the sun on the moon produced a movement of the nodes of the moon's orbit. In other words, a line drawn perpendicular to the plane in which the moon is moving will trace out a cone in $18\frac{1}{2}$ years. He deduced what would be the movement of the nodes of the orbit of a little moon near the earth's surface, revolving in a sidereal day. The same thing would happen, he says, to a ring of moons, whether they did not eventually touch each other or whether they were molten and formed a continuous ring. Next let the ring of moons be attached to the sphere and communicate its motion. Thus the axis of a mass composed of the ring and sphere together will have movement due to the sun of $9''$ annually, and the effect of the moon is in the proportion of the tide-raising force of the two bodies. Airy remarks: "If at this time we might presume to select the part of the '*Principia*' which probably astonished and delighted and satisfied its readers more than any other, we should fix, without hesitation, on the explanation of the precession of the equinoxes."

A good deal of space in Book III. is given to comets. Tycho Brahe had shown that they moved in the celestial spaces and did not belong to our atmosphere. Newton regarded them as bodies like the planets, but moving in very elliptic, nearly parabolic orbits, and showed how from three observations their paths might be determined. The tails arise from the atmosphere of the heads, their great length and splendour being due to the heat which the sun communicates to the comet when near to it. He notes that the perihelion distance of some comets is very small, that of 1680 being only one-sixth of the sun's diameter from the sun's surface. Owing to resistance it is possible they may, in course of time, fall into the sun. He hazards the suggestion that fixed stars may be recruited by the fall of comets into them, and here

may be a possible explanation of the new stars which have appeared from time to time.

There are two other contributions of Newton to astronomy to which I should like to make a brief reference. In the "System of the World," a work which Newton intended at one time to form Book III. of the "Principia," there is a little-known estimate of the distance of the fixed stars. As it was one hundred years before any better estimate was obtained, I venture to direct attention to it.² The disc of Saturn, Newton says, is 17" or 18" in diameter. It receives $1/2,100,000,000$ th of the sun's light. If we suppose Saturn to reflect $\frac{1}{4}$ of the light it receives, the whole light reflected from its illuminated hemisphere will be $1/4,200,000,000$ th of the light emitted from the sun's hemisphere. If the sun were $10,000\sqrt{42}$ times more distant than Saturn, it would yet appear as lucid as Saturn now does without its ring. Let us, therefore, suppose that the distance from which the sun would shine as a fixed star to exceed that of Saturn by about 100,000 times. This gives 0".2 as the annual parallax of a first magnitude star and 0".002 as its diameter. Newton answers the objection that light may be absorbed in its passage through space, for in that case the remoter stars would scarcely be seen.

² Schlesinger, *Publ. Astr. Soc. Pacific*, vol. 14, p. 170.

We may also recall the fact that Newton was the first person who actually constructed a reflecting telescope. He gives an account in the *Phil. Trans. Roy. Soc.* of the chemical experiments he made for the best composition of speculum metal. The use of pads of pitch on which rouge is spread for polishing mirrors and lenses was invented by him. I believe instrument-makers have found no better materials since.

Returning to Book III. of the "Principia," nowhere has so much been comprehended in so few words as in the law of gravitation. It does not explain to us why the moon and planets move as they do, but it does show us how they move and enable us to calculate their movements. Generations of mathematicians and astronomers have employed large parts of their lives in tracing its consequences in most intricate details, comparing the results with observations, and discerning the past and future history of the solar system. Laplace, perhaps the most illustrious of Newton's followers, says "the number and generality of Newton's discoveries relative to the system of the world, the multitude of original and profound views, which have been the germ of the most brilliant theories of the geometers of the last century, will assure to the 'Principia' a pre-eminence above all other productions of the human intellect."

Newton's Work in Mechanics.¹

By Prof. HORACE LAMB, F.R.S.

IF we wish to realise the extent and importance of Newton's contributions to the science of Dynamics, the most obvious way is to compare the long series of propositions contained in the first two books of the "Principia" with the brief record of what had been accomplished by his predecessors and contemporaries. To both he makes ample acknowledgment. Galileo, freeing himself from scholastic tradition, and appealing to experiment as the ultimate test, had ascertained the laws of falling bodies and, assuming the composition of motions, had deduced the parabolic path of a projectile. Considering, further, motion on a smooth incline, he had proved that the velocity acquired from rest must depend only on the vertical height fallen through, and had supported this theoretical result by an ingenious experiment. It is remarkable that this particular result should have been adopted by Huyghens as the basis of his treatment of the cycloidal pendulum instead of the more natural method of

resolution of forces, which we should expect. Huyghens, for some reason, seems not to have been satisfied with Galileo's demonstration, and argued that if the theorem did not hold it would be possible for a particle descending one incline and ascending another to pass from rest to rest at a higher level. We have here, of course, a partial apprehension of the principle of energy. His work on the compound pendulum, the first and for a long time the only example of what we call Rigid Dynamics, was based on a similar argument, the assumption here being that a system of particles cannot of itself move from rest in such a way as to raise its centre of gravity. Huyghens, again, had published (without proof) his theorems on centrifugal force, and simultaneously with Wren and Wallis had investigated the laws of collision of bodies. He has the distinction of being mentioned by Newton more than once, with especial respect.

This brief outline embraces, I think, almost all that is important from the point of view of pure Dynamics. The advance in knowledge within a period of little more than half a century was, it is

¹ Address delivered on Mar. 19 at Grantham in connexion with the commemoration of the two-hundredth anniversary of the death of Sir Isaac Newton.