The Problem of Stellar Evolution.

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The great problem of the evolution of the stars may be attacked along two main lines. We may study the properties of the stars themselves, as revealed by observation, and find orderly sequences among them; or we may analyse, on general physical principles, the constitution of a mass of gravitating matter, and the probable sequence of its changes.

Advances on these two fronts have shown a certain tendency to alternation. Lockyer's conception of stars of rising and falling temperature was based mainly on general physical considerations. The recognition of the sequences of giant and dwarf stars lent strong support to this theory, and—as the present writer showed some eleven years ago—a great mass of observed details fits in with remarkable completeness with the idea that the stars rise in temperature until the gas in their interior becomes compressible only with difficulty, and then cool down again.

More recently, progress has been mainly on the theoretical side, and has been very rapid. Among the milestones on the way may be noted the application of the theory of radiative equilibrium to the internal constitution of the stars, the appreciation of the fundamental importance of radiation pressure in this equilibrium, and of ionisation in making the mean molecular weight low and almost independent of chemical composition—then, recently, the development of rational, rather than empirical, expressions for the elusive opacity-constant, and the recognition that the dismembered atoms inside the stars are so small that even at enormous densities the material must behave like a perfect gas. Several investigators—Jeans, Kramers, Eggert—have contributed to this field, but much the largest share is Eddington's.

Meanwhile, observation has established conclusive evidence—with the joint help of astronomy, physics, chemistry, geology, and biology—that the life of the sun must be enormously long, and that the stars must have within themselves some vast store of potential energy, of hitherto unimagined extent.

These new developments must obviously lead to changes in the theory of stellar evolution to which reference was made above. One frank, but not unfriendly, critic recently characterised these changes as "sudden death." The writer—remembering Mark Twain's response to the rumour of his own demise—believes that in this case, too, the reports have been "greatly exaggerated."

On one point there can be no possible doubt. The feature of the older theory which assumed a fall of internal temperature in the denser dwarf stars owing to the close-packing of the atoms, must be finally abandoned. Eddington's argument is conclusive, and it is clear that the low surface temperatures of these stars must be ascribed, not to low internal temperatures, but to the increase of opacity with density, which prevents the heat from leaking out quickly to the surface.

The theory of the internal constitution and the luminosity of the stars is now really in a fairly satisfactory state. The relations connecting the mean molecular weight and the opacity-constant with the temperature and density appear to be well enough known to assure us that the approximations used by Eddington and Jeans must be close to the truth. Only one quantity remains uncertain—η, which represents the ratio of the average rate of generation of heat per gram in the portion of the star within a given distance of the centre to the corresponding quantity for the whole star—and Eddington has just shown that great changes in the law of its increase toward the centre affect the surface characteristics of the star but little.

Though approximations must be made in the solution of the equations, and opinions differ as to which is best, the main results are clear. The luminosity of a star (its total radiation) increases rapidly with increasing mass, but changes relatively little with the surface temperature, so that the influence of the latter may be expressed by a subsidiary correction—which rarely, if ever, reaches one magnitude, if the solar type is taken as standard.

The new theory, therefore, indicates that a star of given mass must be not far from a definite absolute magnitude, but may have any radius, surface temperature, and spectral type (the old restriction to densities less than a certain limit being unfounded).

The first of these conclusions is strikingly confirmed by observation, both for the most accurate individual data and for averages covering all the available material.

The second, however, is in definite disagreement with the facts. The stars of a given mass—or a given absolute magnitude, which are far easier to pick out, and afford an equally good test of the theory—are by no means indiscriminately distributed among the various spectral classes.

Among the brightest stars, it is true, all spectra are found; but among stars of not more than ten times the sun's luminosity, a large majority of those of given brightness are found within narrow spectral limits. Observational selection is much less disturbing if the grouping is made in this way, and there can be no doubt of the reality of the phenomenon.

From this viewpoint the stars may be divided into three groups:

1. The main sequence (a name suggested by Prof. Eddington), for which the luminosity diminishes rapidly, with increasing redness. This sequence includes most of the O, B, and A stars and all the ordinary dwarfs, and represents the most pronounced axis of concentration of the points upon the familiar diagram in which absolute magnitude is plotted against spectral type.
2. The giants—lying on the bright side of the main sequence, with representative points widely scattered, but showing a fairly definite axis of concentration, joining the main sequence near class F, and running somewhat upward for the redder stars.
3. The white dwarfs, of low luminosity and high surface temperature. Few such stars are yet known, but there are three of them within six parsecs, and they must be more abundant per unit volume of space than any other class except the K and M dwarfs.

Fig. 1 shows the relations for twenty typical stars (taken mainly from Eddington's list). Nos. 6 to 18 clearly indicate the main sequence. Nos. 1 to 5 give a sketchy idea of the giants, and 19 and 20 are the most notable white dwarfs.

To account for this distribution, something more than the internal equilibrium of the stars must be considered. The problem is intimately bound up with that of the source of stellar energy, and the probable secular diminution of stellar mass.

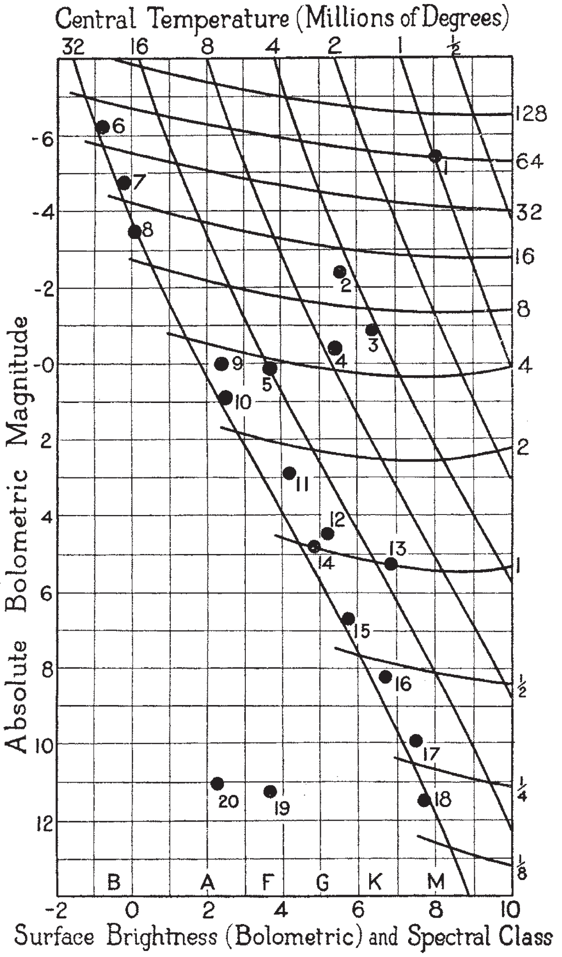

Fig. 1.—1, Antares; 2, δ Cephei; 3, Arcturus; 4, Capella A; 5, Capella B; 6, Plaskett's star; 7, V Puppis; 8, Y Cygni; 9, β Aurigæ; 10, Sirius A; 11, Procyon; 12, α Centauri A; 13, α Centauri B; 14, sun; 15, ξ Bootis A; 16, ξ Bootis B; 17, Kruger 60 A; 18, Kruger 60 B; 19, Sirius B; 20, O_2 Eridani B.

All commentators agree that if the mass of a star remains nearly constant throughout its history, no comprehensive scheme of evolution appears to be possible. But, if the major part of the mass can ultimately be transformed into energy and radiated away, the problem becomes more hopeful.

The first question to be considered is whether the rate of transformation of matter and generation of heat within a star is independent of the temperature and pressure, or not. If the former is true, the star must expand or contract until the rate of loss of heat from the surface balances the unalterable income; if the latter, until the rate of production balances the loss.

An accumulation of heat inside a gaseous star compels its expansion. Unless this is accompanied by an increase of the loss of heat from the surface, a star of which the internal income of heat is fixed cannot be in stable equilibrium. Even if the outgo balanced the income at the start, the slightest deviation would go on increasing, until the star either expanded indefinitely or contracted to minute size. Now recent theory indicates that it is very probable that increase in diameter, and fall of surface temperature, go with decrease in total radiation. Hence, as Eddington points out, the theory that the rate of generation of heat is independent of the internal conditions appears to be untenable.

If this rate varies with the temperature (or perhaps with the density) it is practically certain that it must increase rapidly as the temperature rises—for there is certainly no considerable generation of heat inside the earth. In this case the expansion of the star lowers the internal temperature, and cuts off the excess supply of heat; and the adjustment to such a condition that just enough heat is generated to supply the leakage to the surface will be automatic and stable.

It appears, therefore, necessary to conclude that the rate of transformation of matter into energy increases with the temperature. The thermodynamic difficulties in the way of this hypothesis are serious, but probably not insuperable.

Here, however, we can no longer call our present knowledge of the general properties of matter to aid in solving the specific problem; rather we must once more be guided by the observed astrophysical data.

The two sets of curves in Fig. 1 are computed from Eddington's theory (Monthly Notices, R.A.S., 84, 104, and 308, 1924) (taking fuller account of the probable change of molecular weight) and represent stars of fixed mass or fixed central temperature.

It is at once obvious that all the stars of the main sequence have very nearly the same central temperature—about thirty million degrees. A theory based on different approximations might not make these temperatures come out so remarkably alike, but would still leave them very similar. The giants are cooler inside, and the white dwarfs must be hotter, though numerical calculations are unsafe for such great densities.

The concentration of stars along the main sequence can now be simply explained by assuming that, in the neighbourhood of a temperature of about thirty million degrees, the rate of transmutation of matter into energy increases very rapidly. A higher central temperature than this would generate heat faster than it could escape, the star would expand and cool, and, practically, it could not pass this limit. One would expect the internal temperature to be somewhat higher for the stars of great luminosity (which radiate more heat per gram). An entirely permissible change in the constants of the theory would allow this.

If all the matter in the stars behaved in this way we should expect a star to pass through the giant stages precipitately, since the generation of heat at lower internal temperatures would not suffice for equilibrium. This is evidently not the case, so that it is necessary to postulate that there are also one or more forms of matter which are transformed at lower temperatures, and supply the "fuel" for the giants. Some highly refractory constituent seems to be indicated by the white dwarfs.

The first stage of a star's history which can be clearly visualised is, then, a sphere of very rarefied gas, comparable in diameter to the orbit of Uranus or Neptune, and with a central temperature of a few hundred thousand degrees. Losing heat by radiation, it contracts, at first very rapidly, drawing upon its gravitational energy. When the central temperature reaches some critical value—probably rather less than a million degrees—the degradation of some form of matter (either wholly into energy or into some less massive form, with corresponding energy-emission) begins, and a star is born. The rate of evolution now depends on that of the exhaustion of the transformable material, and, as this is used up at the centre, the star must slowly contract so that the temperature rises, and new regions, nearer the surface, become the main seat of the transformation.

If, as von Zeipel believes, and Jeans doubts, there is active mixing by radial convection, the region of transformation will be less localised, but the general result the same—the temperature rising, as the quantity of exhaustible material diminishes, in order to keep up the rate of liberation of heat. Several successive processes of this sort, involving the transformation of various kinds of matter with different critical temperatures, may be operative during the giant stage.

We must next suppose that as a temperature of some thirty million degrees is approached, a process comes into play which leads to the actual annihilation of the main mass of the stellar material, with a correspondingly great liberation of energy. The central temperature will then remain nearly constant, and the star steadily decrease in mass, "burning itself away" at the hot centre, gradually growing dense and more opaque, and passing down the main sequence.

Finally, to account for the white dwarfs, we must believe that there exists a certain residue of refractory material, immune to transformation at thirty million degrees. As the main constituents become exhausted this will preponderate, and at last be almost exclusively present. If this residue were incapable of transformation, rapid gravitational contraction would ensue until even the ionised atoms were jammed close together. The considerable abundance of the white dwarfs per unit of volume suggests, however, that further energy-liberating changes occur and delay the last act.

The course of evolution would then be represented, on the familiar diagram, not by a reversed figure 7, but rather by a reversed letter *Z*. During the giant stage the surface temperature rises, and the representative point moves to the left. Then along the main sequence it falls, and the point moves downward and to the right. Finally, the star becomes a white dwarf, the temperature rises, and the point moves to the right again.

Whether the lines representing the giant and the white dwarf stages run nearly horizontally, or downward to the right, depends upon the loss of mass in these stages—that is, upon the quantity of matter available for transformation within the corresponding ranges of temperature. The large number of the giants suggests that a considerable fraction of a star's mass is lost during these stages, and that the evolutionary line slopes downward.

If the easily transformable material becomes nearly exhausted before the main transformation sets in, there will be a period of relatively rapid change between the redder giants and the main sequence, and such intermediate stars will be statistically infrequent, as appears to be actually the case.

Too few white dwarfs are known to justify a similar discussion.

It should be especially noticed that while, on the theory here proposed, all stars should pass along at least a part of the main sequence, they may be very different in the other stages. A star of great initial mass would give an evolutionary line crossing the diagram near its top, and joining the main sequence at class B or class A. One of small mass might come in at F or G, or even lower. The existence of such stars, intermediate between the redder giants and dwarfs, is therefore no argument against the theory either in its present form, or for that matter, in the older and abandoned one.

The fainter component of α-Centauri—as Fig. 1 shows—seems to be a star of this sort, and, from the present viewpoint, would be classed as a giant of small mass (as must also be the large red companions of such eclipsing variables as U Cephei).

Similarly, the level at which a star would break away from the main sequence, and become a white dwarf, would depend on the quantity of "refractory" matter originally present (or perhaps formed as a by-product of other transformations), and any combination of absolute magnitude and spectral type is possible.

This scheme of stellar evolution is very similar to its predecessor. The only important point of difference is (as Eddington puts it) "that the diminishing brightness in the dwarf series is due to decreasing mass, and not to a falling off in compressibility." On the other hand, the difference of mass between giants and dwarfs is now explained, and the white dwarfs—formerly most puzzling—now, thanks to Eddington, find an orderly place at the end of the sequence.

The notion of the transmutation of mass into energy, upon which the new theory is founded, appears to rest upon strong evidence. The specific types of transformation postulated above are frankly adopted *ad hoc*, as indicated by the observed statistical facts; but while the subject is still outside the range of existing theories of the constitution of matter there is nothing else to do; and so far as can be judged, the postulates seem plausible enough.

The youngest known stars (not in years, but in evolution) appear to be those of class N and Me, and it is probably no accident that practically all of these are variables. The latest known stage, judging by the density, is found in the companion of Sirius. This is abnormally faint for its mass, and it may be, as Eddington suggests, that close-packing of the atoms is beginning to have an influence here (at a central density of the order of one ton per c.c.!).

The final stage is still uncertain. Either the loss of mass continues indefinitely until the star practically disappears, or else close-packing halts the rise of temperature, before the most refractory atoms can be annihilated. In the latter case the star must ultimately cool down.

There seems little reason to suppose that it would ever return to a " normal " density. Ordinary matter, subjected when cold to increasing hydrostatic pressure, should break down to a state of far smaller volume in which the valence electrons wander freely, while the complete shells of electrons normally inside these approach almost to contact as soon as the work done by the compression exceeds the ionisation energy required to tear the valence electrons loose.

Further—and very great—increases of pressure should break down the *N*, *M*, *L*, and ultimately the *K* shell, and reduce matter to a formless state. Probably the greatest pressure which exists even in a white dwarf would not complete this process except for the lightest atoms ; but the maintenance of a very high density, even if all the heat had been lost, appears entirely possible. Only the surface of such a body would be in the ordinary solid state ; that of the interior would transcend our experience, but not our imagination.

Regions of Tension and Continental Drift.[1]

By Dr. J. W. EVANS, F.R.S.

IF the sima magma, when still in its original position under the heavy pressure of the superincumbent sial, remained at too high a temperature for crystallisation to take place, the magma and the rocks differentiated from it would be of the normal types. The temperature-gradient, however, in the great masses of sial (the continental shields of some authors) shows a relatively slow increase in depth ; consequently, a comparatively low temperature may be found at their under surface, a temperature sufficiently low for some crystallisation to take place.

In these circumstances it may be expected that crystallisation will commence with those minerals or groups of minerals that have a small molecular volume ; small, that is to say, for the elements which they contain. Corresponding to these there are usually minerals or combinations of minerals having the same chemical composition but greater molecular volumes, which crystallise under low pressures. Among the high-pressure minerals with small molecular volumes are garnet, zoisite, epidote, kyanite, muscovite, biotite, and diamond. The low-pressure minerals with large volumes include anorthite, orthoclase, andalusite, and graphite. Albite and water have relatively large volumes, but have for practical purposes no small-volume representatives.

Dr. F. Becke has shown [10] that igneous rocks, formed originally at moderate depths, have, when subjected to great pressure, certain minerals changed into others with greater density : for example, orthoclase into muscovite and quartz, and lime-soda plagioclases into albite and zoisite. He gives the following equation :

$$\underbrace{\underset{\text{albite}}{x(\mathrm{NaAlSi_3O_8})}+\underset{\substack{\text{anorthite}\\402{\cdot}8}}{4(\mathrm{CaAl_2Si_2O_8})}}_{\text{plagioclase}}+\underset{\substack{\text{orthoclase}\\108{\cdot}3}}{\mathrm{KAlSi_3O_8}}+\underset{\substack{\text{water}\\36}}{2\mathrm{H_2O}}$$

$$=\underset{\text{albite}}{x(\mathrm{NaAlSi_3O_8})}+\underset{\substack{\text{zoisite}\\275{\cdot}8}}{2(\mathrm{HCa_2Al_3Si_3O_{13}})}$$

$$+\underset{\substack{\text{muscovite}\\141{\cdot}1}}{\mathrm{H_2KAl_3Si_3O_{12}}}+\underset{\substack{\text{quartz}\\45{\cdot}6}}{2\mathrm{SiO_2}}$$

The figures under the names of the mineral substances other than albite, which is unaltered, express their molecular volumes. It will be found that the total of these volumes under low pressure is 547·1, and under high pressure only 462·5.

When, therefore, plagioclase is subjected to sufficiently high temperature and pressure in the presence of a little water, and is decomposed in the manner described by Becke, the albite portion appears to go into temporary solution, and either recrystallises as water-clear albite *in situ*, or is the agent of the albitisation of neighbouring rocks, giving rise to the formation of spilites, spilosites, desmoisites, or adinoles. It is reasonable to suppose that the minerals which crystallise out from amorphous magmas under heavy pressure will be similar to those into which other minerals already crystallised are transformed under similar conditions of pressure and temperature.

It is, however, the garnets which are chiefly characteristic of rocks that crystallise under especially heavy pressure. Dr. L. L. Fermor [11] has described the occurrences of rocks in the Province of Vizagapatam (India), which appear to have been crystallised under such conditions.

These rocks are characterised by the presence of garnets which, under normal conditions of crystallisation, would have been replaced by anorthite, augite, diopside, hedenbergite (iron-diopside), wollastonite, olivine, tephrite (manganese-olivine), and magnetite. The garnetiferous rocks are calculated by Dr. Fermor to occupy 20 per cent. less space than their low-pressure equivalents. They contain orthoclase, although this, as we have seen, may be transformed under special conditions of pressure into muscovite and quartz, but they contain no soda-minerals. Presumably, under the conditions that prevailed, these would remain in the fluid state the longest, and only crystallise when the temperature had been lowered still more. Apparently they were pressed out in the fluid state from the Vizagapatam rocks, on a release of pressure, before this happened. These rocks appear to be made up of minerals which have crystallised out at great pressure below the under surface of the sial crust. Their crystallisation must have left a magma rich in soda and comparatively poor in the oxides of divalent minerals, such as lime, magnesia, and ferrous and manganous oxides.

We may expect a magma of this character to occur, either alone or in association with typical sima magma, in the molten magma arising from great depths in fissures formed in regions of tension. Such a magma

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[1] Continued from p. 175.

[10] " Ueber Beziehungen zwischen Dynamometamorphose and Molekularvolumen," *Neues Jahrb.*, vol. 2 (1896), pp. 182-83.

[11] " Garnet as a Geological Barometer and an Infra-Plutonic Zone of the Earth's Crust," Rec. Geol. Surv., India, vol. 42 (1913), pp. 41-47.