A New Determination of the Distribution of Stars with respect to Magnitude and Galactic Latitude.

By FREDERICK H. SEARES and Prof. P. J. VAN RHIJN.

THE significance of the numbers of stars visible in different parts of the sky has been recognised since the time of Sir William Herschel, whose star gauges first indicated the flattened, watch-shaped form of the galactic system. The stars having parallaxes which can be measured, either by trigonometric or spectroscopic methods, are all so near that their distances tell nothing of the structural features of the system. These must be learned from counts, like those of Herschel, of stars of different magnitudes in different parts of the sky.

Were the stars all equally luminous, it would be easy to find their distribution in space. Differences in apparent brightness would then be wholly an effect of differences in distance, and star-counts, together with the parallax of a single star, would determine the distances of all objects within reach of our telescopes. Actually, the range in stellar luminosity is enormous, at least 100,000,000 to 1. Differences in luminosity, however, can be allowed for, provided we can find the numbers of stars of different intrinsic brightness in a given volume of space;¹ but this problem has not yet been satisfactorily solved.

Investigations by Kapteyn and van Rhijn indicated that the distribution of luminosity could be represented by a Gaussian error curve, with a maximum frequency for luminosities 0.07 that of the sun. The present authors, however, have independently shown that this maximum is not real and that the frequencies continue to increase probably at least so far as luminosities equal to 0.001 that of the sun. The highly luminous stars, on the other hand, are well represented by the ascending branch of an error curve. Thus at present we have an excellent approximation for the relative numbers of stars intrinsically brighter than the sun, which stands about midway in the known range of luminosity, but only general indications as to the behaviour of fainter stars. This somewhat restricts the immediate usefulness of star-counts but does not lessen their ultimate value.

The star gauges of Sir William Herschel were extended to the southern hemisphere by Sir John Herschel in 1834-38, and the accumulation of star-counts thus begun has been continued by others, in recent years especially by Kapteyn and van Rhijn, whose respective distribution tables are in "Groningen Publications," Nos. 18 and 27. Successive revisions of the data have extended the counts to larger areas of the sky and to lower limits of brightness, and have also improved the scale of magnitudes to which the counts are referred.

The serious practical difficulty lies in the magnitude scale, which must be accurate if the counts are to be useful. The range in apparent brightness covered by large telescopes is 20 magnitudes or more. The establishment of standards over this interval requires the ultimate comparison of light sources the intensities of which are in the ratio of at least 100,000,000 to 1. Under the most favourable conditions, sources of the

same order of brightness and of the same colour can be compared with an uncertainty of about 1 per cent.; but as the differences in brightness and spectral composition increase, the uncertainty becomes much larger. Comparable difficulty in the measurement of lengths would mean, for example, an error of several inches in determining the dimensions of a room. An ordinary yardstick does a hundred times better than this, let alone the precise methods of laboratory measurement.

The difficulty of bridging a long interval of brightness by visual methods is illustrated by the scale underlying Kapteyn's table of stellar distribution. This, the best visual scale available up to 1915, is now known to be a magnitude in error at the sixteenth magnitude, the equivalent of 150 per cent. in the light-intensity of the stars to which this magnitude was assigned. Mainly because of this same difficulty the conspicuous increase in the concentration of stars in the Milky Way with increasing magnitude, which was already indicated by the gauges of Sir John Herschel, remained an open question until 1917, when counts of 40,000 stars observed photographically at Mount Wilson put the matter beyond doubt.

The application of photographic methods by Pickering and Miss Leavitt did much to increase the precision, and the resulting Harvard photographic scale, defined by a sequence of stars at the North Pole, is the basis of the distribution table of van Rhijn. Investigations in photographic photometry begun at Mount Wilson in 1910 led to the Polar standards of brightness published by Seares in 1915. These were afterwards strengthened by additional data, and to the sixteenth magnitude have been confirmed at several observatories; below the sixteenth magnitude, however, the Mount Wilson standards are still the only ones available.

These investigations, which comprise the Mount Wilson contribution to the International Scale adopted at Rome in 1922, were preparatory to a study of the distribution of faint stars in the 139 Selected Areas of Kapteyn between the North Pole and declination -15° , an undertaking now finished. The individual magnitudes will be published in the "Mount Wilson Catalogue of Selected Areas." The detailed discussion of the counts based on these magnitudes and a revision of certain earlier results will also be published elsewhere. Some matters of general interest, however, may be summarised here.

The new distribution table depends on: (a) The Mount Wilson "Catalogue," which completely determines the distribution for stars between magnitudes 13.5 and 18.5; (b) Van Rhijn's tables, "Groningen Publication," No. 27, reduced to the international scale by (a). These afford results for the interval $m=4.0$ to 9.0 and establish the scale for counts from a third source, (c), consisting of 33 zones of the Astrographic Catalogue between declinations -65° and $+62^\circ$. These counts, published by Turner, include about 1,400,000 stars and determine the distribution in the interval $m=9.0$ to 13.5.

The Mount Wilson "Catalogue" (a) is based on two partially independent investigations: (a_1) measures

¹ Practically we are obliged to assume that the relative numbers are the same at all distances.

made at Mount Wilson on photographs of 15 minutes exposure or less, which establish the scale and give the magnitudes of 65,683 stars in fields 23' in diameter; and (a_2) measures at Groningen on Mount Wilson photographs of 60 minutes exposure. These give the magnitudes of 44,910 stars in fields 15' $\times$ 15' or 20' $\times$ 20', based on standards derived from (a_1). Most of these stars are included in (a_1), so that the total number of individual objects is approximately 70,400. The magnitudes are on the international scale.

The fields in the 139 Selected Areas cover but 1/2500 of the entire sky. Nevertheless, examination shows that had the alternate areas been discussed separately, the two series of counts would have led to nearly identical results. Individual areas show large deviations from the mean, but with the exception of regions in the Milky Way there is little systematic uncertainty. This perhaps as well as anything illustrates the underlying statistical unity of the stellar system. Properly selected, 1/5000 of the stars between magnitudes 13.5 and 18.5 reveal the main features of the distribution with only a small percentage of error.

The densities derived from the combination and adjustment of data for the three intervals 4.0-9.0, 9.0-13.5, and 13.5-18.5 are so regular that they have been extrapolated to the twenty-first magnitude, which is the practicable limit for long-exposure photographs with the largest telescopes. The table, which thus covers a range of 17 magnitudes, gives the logarithm of the average number of stars per square degree brighter than photographic magnitude m ($\log N_m$) situated in different galactic latitudes. Local irregularities, differences between northern and southern latitudes, and systematic deviations in longitude have been ignored for the present. Averages for latitude intervals 0°-20°, 20°-40°, 40°-90°, and for the whole sky are also given. Correction of the magnitude limits for colour leads to average densities for the entire sky for a grouping of stars according to *visual* magnitude. The complete transformation of the table to visual limits cannot be made at present because of lack of data for the variation of mean colour with galactic latitude.

The total number of stars to the twenty-first photographic magnitude is 8.9×10^8 , or, to the twentieth visual magnitude, a round 10^9 . It is difficult to specify the uncertainty attached to these numbers. There is no means of testing the magnitude scale for the very faint stars; but if the limiting magnitude of the table is correct within one or two tenths, we may consider ourselves fortunate. Since the average change in N_m per magnitude at the limit is about one-half of N_m itself, the uncertainty in the totals from this source alone can scarcely be less than 5 or 10 per cent. Errors in the final counts arising from the choice of regions used as samples, as already stated, must be small, except possibly in low galactic latitudes. The fields in the Milky Way are not numerous, nor are they well distributed in galactic longitude, and the tabular densities are here subject to considerable uncertainty, although the amount cannot be stated.

For bright stars at all points in the sky, the ratio of totals to successive magnitude limits N_{m+1}/N_m is 2.9. With decreasing brightness this ratio falls off, but most rapidly in the direction of the galactic

poles. Thus in passing from the twentieth to the twenty-first magnitude the total in the Milky Way is increased 1.8 times, while at the poles the increase is only 1.4 fold. Were the stars uniformly distributed in space, the ratio (excluding losses of light by absorption, scattering, etc.) would be constant and equal to 3.98. The actual ratios therefore show that the stars thin out with increasing distance from the centre; that at great distances they thin out more rapidly than near the sun; and that this thinning out is most pronounced in the direction of the poles of the Milky Way—results obviously related to the flattened, watch-shaped form of the system.

The variation in $\log N_m$ can be used to estimate the probable total of all the luminous stars, both visible and invisible, in any direction, and hence also the total for the system. Earlier data indicated that dN_m/dm could be represented by an error function for practically all values of m to the limit of the counts. It was, therefore, natural to assume that the law also applied to stars too faint for observation, whereupon a simple integration gave the required total. The relation between star-counts and the luminosity and density functions indicates, however, that the values of dN_m/dm for the brighter stars cannot be well represented by an error function, and the present counts confirm this conclusion. Below the twelfth or thirteenth magnitude the representation is excellent, but above this limit the second differences in $\log (dN_m/dm)$ are not constant. This restriction raises a serious question as to whether the error function applies to the invisible stars. For lack of anything better, we assume that it does apply, and thus find: Total number of stars per square degree in galactic latitude 0°, 5,320,000, one-half of which are fainter than magnitude 30.6. For latitude 90° the total is 7160, with one-half fainter than magnitude 23.9. The corresponding totals to $m=21.0$ are 73,600 and 1667 respectively. Hence in the direction of the poles of the Milky Way about one-fourth of the stars between the sun and the limits of the system are accessible to observation. In the direction of the Milky Way the fraction is one-seventieth.

Stars of all magnitudes are most numerous in the Milky Way, but the concentration in the galaxy is obviously much greater for faint than for bright stars. At the fourth magnitude the ratio of the values of N_m for 0° and 90° is 3.5; at the twenty-first magnitude it is 44; to the limits of the system, 743. The importance of the Milky Way as a structural feature of the system is also indicated by the fact that 95 per cent. of all the stars are within 20° of the galactic plane. The remaining two-thirds of the sky contain but 5 per cent., of which less than 1 per cent. is in that third between latitudes 40° and 90°.

Three different methods of summation give values for the total in the system ranging from 3.0×10^{10} to 3.7×10^{10} , of which the lower limit is the most probable value. This, however, like all other results depending on the assumption that the adopted formulæ for dN_m/dm hold to the limits of the system, is very uncertain. Even at the galactic poles, where a considerable fraction of the stars can be observed, the extrapolation is risky enough; and naturally it is very much more so in the Milky Way, where apparently

less than 2 per cent. of the stars are within observational reach. But whatever the actual total, it is clear that the number of stars beyond the range of the largest telescopes is many times that accessible to observation.

The total amount of starlight is little affected by questions as to the number of stars in the system. For the distribution given above, the stars brighter than the twentieth visual magnitude, comprising but 3 per cent. of the total, contribute 98 per cent. of all the starlight. The integrated visual light for the

whole sky is the equivalent of 1076 stars of visual magnitude 1.0 on the international scale. The corresponding numbers found by Yntema and van Rhijn from measures of the brightness of the sky are 1350 and 1440, the unit being referred to the Harvard scale. The outstanding difference corresponds to about 0.3 mag. Since the stars of about the twelfth apparent magnitude contribute the largest amount of light, no reasonable correction to the magnitude scale will wholly account for the difference.

The Expedition of the R.R.S. *Discovery*.

THE Royal Research Ship *Discovery*, which is leaving England at the end of the present month, will be engaged for the next two years in oceanographical investigations in the South Atlantic and Antarctic. The ship, which is barque-rigged with auxiliary steam, was built in 1901 for the late Captain Sir Robert Falcon Scott, and was acquired on behalf of the Falkland Islands Government for the purpose of the present investigations in 1923. During the last eighteen months she has undergone extensive repairs, and changes have been made in her masting and sail-plan in accordance with experience obtained by Capt. Scott. She has been refitted throughout, and is now completely equipped for the investigations for which she is intended.

The cost of the expedition will be met entirely from public revenues raised in the Dependencies of the Falkland Islands, and the work will be controlled, subject to the instruction of the Secretary of State for the Colonies, by an executive committee, constituted as follows: Mr. E. R. Darnley (Colonial Office, chairman); Sir Sidney Harmer (British Museum, vice-chairman); Sir J. Fortescue Flannery, Bt. (consulting naval architect); Mr. H. T. Allen (Colonial Office); Mr. J. O. Borley (Ministry of Agriculture and Fisheries); Capt. J. D. Nares (Admiralty), and Mr. J. M. Wordie (Royal Geographical Society); Mr. H. Horsburgh (technical assistant); Mr. E. W. Baynes (secretary).

The scientific officers of the expedition are: Dr. Stanley Kemp (Director of Research); Messrs. A. C. Hardy, J. E. Hamilton, N. A. Mackintosh, J. E. G. Wheeler, L. H. Matthews, and E. R. Gunther (zoologists); Messrs. H. F. P. Herdman and A. J. Clowes (hydrologists). Messrs. Mackintosh, Wheeler, Matthews, and Clowes are at present serving at the shore station, South Georgia. The marine staff includes: Commander J. R. Stenhouse (captain); Lieut.-Commander W. H. O'Connor (chief officer); Lieut.-Commander J. M. Chaplin (second officer and surveyor); Eng.-Lieut. W. A. Horton (chief engineer), and Lieut.-Colonel E. H. Marshall (surgeon).

The principal object of the expedition is to obtain information bearing on whales, more especially on those species which form the basis of the industry now flourishing at South Georgia and in the South Shetlands. Although the whale fisheries in these places are controlled by Government regulations, very little is known of the habits and migrations of the animals and of the reasons for the fluctuation in their abundance. It is, indeed, not yet certain whether the closely similar Arctic and Antarctic whales are specifically identical. There is, therefore, much information to be acquired before a satisfactory basis for the control of the industry can be found.

In connexion with the investigations, a laboratory has been established in Cumberland Bay, South Georgia, where examination is being made of the whales captured at the adjacent whaling station. At this laboratory good progress has already been made, for, although the building was only completed in February, 181 whales had been examined by April 10.

Work on the *Discovery* will mainly be directed towards obtaining information on oceanographic conditions in the waters frequented by southern whales, and routine observations on the hydrography and plankton will be made throughout the voyage. On the whale feeding-grounds intensive work will be undertaken and a close study made of the euphausians, which occur seasonally in great abundance, and constitute the principal, if not the only food of southern rorquals. Plankton nets will be employed at all depths, the smaller hauled vertically and the larger, up to a diameter of 4½ metres, horizontally. New apparatus for opening and closing horizontal nets will be tried, together with a mechanism designed to allow of a number of vertical nets being operated on a single line. A very large midwater trawl, with three otter boards, 250 feet in length and with a mouth area of about 1500 square feet, will be used in an attempt to obtain large pelagic organisms, particularly cephalopods, which would be able to avoid smaller nets. Depth recorders, of pressure gauge and thermometer types, will be used to check the depths at which towed nets are fishing. For work on the bottom the ship is provided with dredges of various kinds, traps similar to those designed by the Prince of Monaco, the Petersen grab, and both beam and otter trawls, but the latter will ordinarily be used only on the coastal banks where fish in commercial quantities are likely to be found.

The deck equipment for biological and hydrographic work comprises a large trawling winch carrying 5000 fathoms of tapered warp on one reel and 1000 fathoms of trawl warp on the other. An auxiliary reel with 3500 fathoms of 6 mm. wire is driven from this winch. For vertical nets and hydrographic appliances there are four smaller reels, driven by three engines and all fitted with 4 mm. wire. Two of these reels carry 3500 fathoms of wire for deep observations and two carry 500 fathoms for observations at lesser depths. A Lucas sounding-machine is installed with various attachments for the collection of bottom samples, and a dynamometer, accumulators, metre recorders, an electric centrifuge, and other subsidiary apparatus are provided. There are two laboratories on the ship, one on the upper and one on the main deck, completely equipped for biological and hydrographic work.