endoparasitic organisms, will determine largely the extent to which he can use and develop the natural resources of the rich tropical and sub-tropical zone of the earth.

Other applications of zoology to human well-being cannot be dealt with here, but mention should be made of two—the researches on sea-fisheries problems which have formed an important branch of the zoological work of Great Britain for forty years, and the studies on genetics which made possible an explanation of the mode of inheritance of a particular blood-group, and of some of the defects (*e.g.* colour-blindness and hæmophilia) and malformations which appear in the human race.

The Theory of the Affine Field.[1]

By Prof. ALBERT EINSTEIN, For. Mem. R.S.

THE theory of the connexion between gravitation and electromagnetism outlined below is founded on Eddington's idea, published during recent years, of basing "field physics" mathematically on the theory of the affine relation. We shall first briefly consider the entire development of ideas associated with the names Levi-Civita, Weyl, and Eddington.

The general theory of relativity rests formally on the geometry of Riemann, which bases all its conceptions on that of the interval ds between points indefinitely near together, in accordance with the formula [2]

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu \quad . \quad . \quad . \quad . \quad (1)$$

These magnitudes $g_{\mu\nu}$ determine the behaviour of measuring-rods and clocks with reference to the co-ordinate system, as well as the gravitational field. Thus far we are able to say that, from its foundations, the general theory of relativity explains the gravitational field. In contrast to this, the conceptual foundations of the theory have no relations with the electromagnetic field.

These facts suggest the following question. Is it not possible to generalise the mathematical foundations of the theory in such a way that we can derive from them not only the properties of the gravitational field, but also those of the electromagnetic field ?

The possibility of a generalisation of the mathematical foundations resulted from the fact that Levi-Civita pointed out an element in the geometry of Riemann that could be made independent of this geometry, to wit, the "affine relation"; for according to Riemann's geometry every indefinitely small part of the manifold can be represented approximately by a Euclidean one. Thus in this elemental region there exists the idea of parallelism. If we subject a contravariant vector A^σ at the point x_ν to a parallel displacement to the indefinitely adjacent point $x_\nu + \delta x_\nu$, then the resulting vector $A^\sigma + \delta A^\sigma$ is determined by an expression of the form

$$\delta A^\sigma = -\Gamma^\sigma_{\mu\nu} A^\mu \delta x_\nu \quad . \quad . \quad . \quad . \quad (2)$$

The magnitudes Γ are symmetrical in the lower indices, and are expressed in accordance with Riemann geometry by the $g_{\mu\nu}$ and their first derivatives (Christoffel symbols of the second kind). We obtain these expressions by formulating the condition that the length of a contravariant vector formed in accordance with (1) does not change as a result of the parallel displacement.

Levi-Civita has shown that the Riemann tensor of curvature, which is fundamental for the theory of the gravitational field, can be obtained from a geometrical consideration based solely on the law of the affine relation given by (2) above. The manner in which the $\Gamma^\sigma_{\mu\nu}$ are expressible in terms of the $g_{\mu\nu}$ plays no part in this consideration. The behaviour in the case of differential operations of the absolute differential calculus is analogous.

These results naturally lead to a generalisation of Riemann's geometry. Instead of starting off from the metrical relation (1) and deriving from this the coefficients Γ of the affine relation characterised by (2), we proceed from a general affine relation of the type (2) without postulating (1). The search for the mathematical laws which shall correspond to the laws of Nature then resolves itself into the solution of the question : What are the formally most natural conditions that can be imposed upon an affine relation ?

The first step in this direction was taken by H. Weyl. His theory is connected with the fact that light rays are simpler structures from the physical view-point than measuring-rods and clocks, and that only the ratios of the $g_{\mu\nu}$ are determined by the law of propagation of light. Accordingly he ascribes objective significance not to the magnitude ds in (1), *i.e.* to the length of a vector, but only to the ratio of the lengths of two vectors (thus also to the angles). Those affine relations are permissible in which the parallel displacement is angularly accurate. In this way a theory was arrived at, in which, along with the determinate (except for a factor) $g_{\mu\nu}$ other four magnitudes ϕ_ν occurred, which Weyl identified with electromagnetic potentials.

Eddington attacked the problem in a more radical manner. He proceeded from an affine relation of the type (2) and sought to characterise this without introducing into the basis of the theory anything derived from (1), *i.e.* from the metric. The metric was to appear as a deduction from the theory. The tensor

$$R_{\mu\nu} = -\frac{\partial \Gamma^\alpha_{\mu\nu}}{\partial x_\alpha} + \Gamma^\alpha_{\mu\beta} \Gamma^\beta_{\nu\alpha} + \frac{\partial \Gamma^\alpha_{\mu\alpha}}{\partial x_\nu} - \Gamma^\alpha_{\mu\nu} \Gamma^\beta_{\alpha\beta} \quad . \quad (3)$$

is symmetrical in the special case of Riemann's geometry. In the general case $R_{\mu\nu}$ is split up into a symmetrical and an "anti-symmetrical" part :

$$R_{\mu\nu} = \gamma_{\mu\nu} + \phi_{\mu\nu} \quad . \quad . \quad . \quad . \quad (4)$$

One is confronted with the possibility of identifying $\gamma_{\mu\nu}$ with the symmetrical tensor of the metrical or gravitational field, and $\phi_{\mu\nu}$ with the antisymmetrical tensor of the electromagnetic field. This was the course taken by Eddington. But his theory remained incomplete, because at first no course possessed of the advantages of simplicity and naturalness presented

[1] Translated by Dr. R. W. Lawson.
[2] In accordance with custom, the signs of summation are omitted.

itself, for the determination of the 40 unknown functions $\Gamma^{\alpha}_{\mu\nu}$. The following brief statement will serve to show how I have endeavoured to fill in this gap.[3]

If the German capital $\mathfrak{H}$ be a scalar density that depends only on the functions $\Gamma^{\alpha}_{\mu\nu}$, then Hamilton's principle

$$\delta\{\int\mathfrak{H}d\tau\}=0 \quad . \quad . \quad . \quad . \quad (5)$$

supplies us with 40 differential equations for the functions Γ, when we stipulate that during the variation the functions Γ are to be treated as magnitudes independent of each other. Further we assume that $\mathfrak{H}$ depends only on the magnitudes $\gamma_{\mu\nu}$ and $\phi_{\mu\nu}$, and thus write

$$\delta\mathfrak{H}=\mathfrak{g}^{\mu\nu}\delta\gamma_{\mu\nu}+\mathfrak{f}^{\mu\nu}\delta\phi_{\mu\nu} \quad . \quad . \quad . \quad (6)$$

where we have

$$\left.\begin{aligned}\frac{\partial\mathfrak{H}}{\partial\gamma_{\mu\nu}}&=\mathfrak{g}^{\mu\nu}\\ \frac{\partial\mathfrak{H}}{\partial\phi_{\mu\nu}}&=\mathfrak{f}^{\mu\nu}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (7)$$

At this point it should be noticed that in the theory developed here, the small German letters respectively represent the contravariant density ($\mathfrak{g}^{\mu\nu}$) of the metrical tensor, and the contravariant tensor density ($\mathfrak{f}^{\mu\nu}$) of the electromagnetic field. Thus in a well-known manner is given the transition from tensor densities (expressed by German letters) to contravariant and covariant tensors (expressed by the corresponding italic letters), and a metric is introduced which rests exclusively on the affine relation.

By performing the variation we obtain after some amount of calculation

$$\Gamma^{\alpha}_{\mu\nu}=\tfrac{1}{2}g^{\alpha\beta}\left(\frac{\partial g_{\mu\beta}}{\partial x_{\nu}}+\frac{\partial g_{\nu\beta}}{\partial x_{\mu}}-\frac{\partial g_{\mu\nu}}{\partial x_{\beta}}\right)-\tfrac{1}{2}g_{\mu\nu}i^{\alpha}+\tfrac{1}{6}\delta^{\alpha}_{\mu}i_{\nu}+\tfrac{1}{6}\delta^{\alpha}_{\nu}i_{\mu} \quad (8)$$

where

$$\frac{\partial\mathfrak{f}^{\mu\nu}}{\partial x_{\nu}}=\mathfrak{i}^{\mu} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Equation (8) shows that our extension of the theory, which appears to be so general, leads to a structure of the affine relation that does not deviate more strongly from that of the geometry of Riemann than is required by the actual structure of the physical field.

We now obtain the field equations in the following manner. From (3) and (4) we first derive the relations

$$\gamma_{\mu\nu}=-\frac{\partial\Gamma^{\alpha}_{\mu\nu}}{\partial x_{\alpha}}+\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha}+\tfrac{1}{2}\left(\frac{\partial\Gamma^{\alpha}_{\mu\alpha}}{\partial x_{\nu}}+\frac{\partial\Gamma^{\alpha}_{\nu\alpha}}{\partial x_{\mu}}\right)-\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} \quad (10)$$

[3] Herr Droste of Leyden hit upon the same idea independently of the present writer.

$$\phi_{\mu\nu}=\tfrac{1}{2}\left(\frac{\partial\Gamma^{\alpha}_{\mu\alpha}}{\partial x_{\nu}}-\frac{\partial\Gamma^{\alpha}_{\nu\alpha}}{\partial x_{\mu}}\right) \quad . \quad . \quad . \quad (11)$$

In these equations the $\Gamma^{\alpha}_{\mu\nu}$ on the right-hand side are to be expressed by means of (8) in terms of the $\mathfrak{g}^{\mu\nu}$ and $\mathfrak{f}^{\mu\nu}$. Moreover, if $\mathfrak{H}$ is known, then on the basis of (7) $\gamma_{\mu\nu}$ and $\phi_{\mu\nu}$, *i.e.* the left-hand sides of (10) and (11), can also be expressed in terms of $\mathfrak{g}^{\mu\nu}$ and $\mathfrak{f}^{\mu\nu}$. This latter calculation can be simplified by means of the following artifice. Equation (6) is equivalent to the statement that

$$\delta\mathfrak{H}^{*}=\gamma_{\mu\nu}\delta\mathfrak{g}^{\mu\nu}+\phi_{\mu\nu}\delta\mathfrak{f}^{\mu\nu} \quad . \quad . \quad . \quad (6a)$$

is also a complete differential, so that if $\mathfrak{H}^{*}$ is an unknown function of the $\mathfrak{g}^{\mu\nu}$ and $\mathfrak{f}^{\mu\nu}$, the following relations will hold:

$$\left.\begin{aligned}\gamma_{\mu\nu}&=\frac{\partial\mathfrak{H}^{*}}{\partial\mathfrak{g}^{\mu\nu}}\\ \phi_{\mu\nu}&=\frac{\partial\mathfrak{H}^{*}}{\partial\mathfrak{f}^{\mu\nu}}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (7a)$$

We now have only to assume $\mathfrak{H}^{*}$. The simplest possibility is obviously

$$\mathfrak{H}^{*}=-\frac{\beta}{2}f_{\mu\nu}\mathfrak{f}^{\mu\nu} \quad . \quad . \quad . \quad . \quad (12)$$

In this connexion it is interesting that this function does not consist of several summation terms which are logically independent of each other, as was the case with the theories hitherto proposed.

In this way we arrive at the field equations

$$R_{\mu\nu}=-\kappa[(\tfrac{1}{4}g_{\mu\nu}f_{\sigma\tau}f^{\sigma\tau}-f_{\mu\sigma}f^{\sigma}_{\nu})+\gamma f_{\mu}f_{\nu}] \quad . \quad (13)$$

whereby $R_{\mu\nu}$ is the Riemann tensor of curvature. κ and γ are constants, f_{μ} is the electromagnetic potential, which is connected with the field strength by the relation

$$f_{\mu\nu}=\frac{\partial f_{\mu}}{\partial x_{\nu}}-\frac{\partial f_{\nu}}{\partial x_{\mu}} \quad . \quad . \quad . \quad . \quad (14)$$

and with the electrical current density by the relation

$$\mathfrak{i}^{\mu}=-\gamma\mathfrak{g}^{\mu\sigma}f_{\sigma} \quad . \quad . \quad . \quad . \quad (15)$$

In order that these equations may be in accord with experience, the constant γ must be practically indefinitely small, for otherwise no fields would be possible without noticeable electrical densities.

The theory supplies us, in a natural manner, with the hitherto known laws of the gravitational field and of the electromagnetic field, as well as with a connexion as regards their nature of the two kinds of field; but it brings us no enlightenment on the structure of electrons.

Further Determinations of the Constitution of the Elements by the Method of Accelerated Anode Rays.[1]

By Dr. F. W. ASTON, F.R.S.

BY further use of the method of accelerated anode rays, results have been obtained with a number of elements since the publication of the isotopes of copper (NATURE, Aug. 4, p. 162). Details of the method will be published later. Most of the following results were obtained by the use of fluorine compounds of the elements investigated.

The mass-spectrum of strontium shows one line only, at 88. This was obtained in considerable intensity. If any other constituents exist they must be present

[1] A paper read on September 18 before Section A of the British Association Meeting at Liverpool.