Definitions and Laws of Motion in the "Principia."

By Sir GEORGE GREENHILL.

MACH'S "Historical Lectures on Mechanics in its Development" ought to have a great influence on the treatment of the subject, with an English version from the Open Court Company. Mach has many a shrewd deep criticism to make on Newton's "Principia," and the present remarks are intended as an amplification of some of his scientific animadversion. It would have been worth his while to examine the previous state of the theory of dynamics to see what laws were current before the statement as given by Newton. These laws must have been enunciated, not only to give precision to the subject, but at the same time to correct and contradict previous fallacy and error, and it would be valuable to have a record in historical development.

The First Law must have excited incredulity, as contradictory to common observation of a body in motion, soon coming to rest of itself; and when the heavenly bodies were pointed at, a divine Primum Mobile was postulated to keep them going eternally, in the pious reflections of Aristotle, quoted in the former conventional manner at the end of the "Principia," and suggesting Napoleon's criticism of Laplace.

Axiomata sive Leges Motus. Lex I. Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

Similar statements can be traced in the writings of Aristotle and Plutarch. But it would be more instructive if we were told something of previous ideas contradicted in this Law, such as "A body in motion will come to rest of itself," as in observation of daily life, ignoring the reason and cause.

Lex II. Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.

A vector change of momentum, *motus*, is indicated here; but the Law requires amplification in a commentary-corollary. *Motus* is quantity of motion, called momentum to-day, and *mutationem motus* requires to be qualified as time change, rate of change per time change, per unit of time; not per length or distance, which would imply energy or *vis viva*, an idea not extant in Newton's day.

Translated into our algebraical symbols, quantity of matter, *quantitas materiae*, of Definition I is denoted by W , lb. (in French it would be denoted by P , kg., for *poids*, *pondus*). Here W , the *Pondus* of our *Corpus*, is measured by weighing it in the scales, corrected for buoyancy of the air, and this is an operation susceptible to the greatest accuracy in physical measurement.

The velocity is v , in f/s (feet per second), so that the quantity of motion is Wv , lb.-ft. per sec., according to Definition II. Velocity v is not so easy to measure to equal accuracy as W .

Then, according to Law II, quantity of motion Wv acquired (from rest) under a force F acting for t seconds, is proportional to Ft (called the impulse), and expressed in a proportion, $Wv \propto Ft$, leaving the unit of force to choice.

The absence of the algebraical sign of equality, $=$, will be noticed as not employed in the "Principia." But in any numerical calculation, equality must be introduced by the appropriate constant factor.

Working with the practical gravitation Unit of Force, the only one in use up to fifty years ago, and still the only one capable of exact measurement, and taking our unit as the gravitation heft of a pound weight, the sign of variation, $\propto$, is replaced by the sign of equality, $=$, in the variation above by introducing g in the right place, and writing it in a homogeneous form

$$(1) \quad W \frac{v}{g} = F t$$

(lb.) (sec.) (lb.) (sec.)

so as to verify when $F=W$, with $v/g=t$, $v=gt$, as in a free vertical fall of the body; $t=v/g$ being the number of seconds of descent to acquire velocity v , ft./sec.; in most practical problems it is near enough to take $g=32$, ft./sec.², in round numbers.

Then if s feet is the distance required to get up speed v from rest in t seconds, the average velocity

$$(2) \quad \frac{s}{t} = \frac{1}{2}v;$$

and multiplying into equation (1)

$$(3) \quad W \frac{v^2}{2g} = F s$$

(lb.) (ft.) (lb.) (ft.)

and $\frac{v^2}{2g} = s$ in a free fall, where $F=W$.

And in a flying start, from velocity u ,

$$(1)* \quad W \frac{v-u}{g} = Ft, \quad (2)* \quad \frac{s}{t} = \frac{1}{2}(v+u), \quad (3)* \quad W \frac{v^2-u^2}{2g} = Fs.$$

With these three equations, (1), (2), (3), any two of which imply the third, the young engineer may carry on for a long time in the linear dynamics, seen on the road and railway or in the air, up and down hill, getting up speed and checking it again with the breaks.

After that a variable force F may be introduced, as in Hooke's Law of the spring; a vibratory motion investigated, shown off in the pendulum, and seen in reciprocating masses of machinery, or a carriage body on springs. Here is theory enough to keep him going for a year.

Then after linear dynamics comes uniplanar dynamics, and the notion of rotation is introduced. A familiar illustration is always at hand in the door; every room has a door. The muscular sense of starting and stopping the rotation can be exercised; also in brandishing a stick, poker, bat, or club.

Angular inertia then requires measurement, although not mentioned in the "Principia," not even in "De motu corporum pendulorum" in Book II, or "In Horologiis et similibus instrumentis, quae ex rotulis commissis constructa sunt," where the *Corpus* may be the compound pendulum of a clock oscillating about its axle. Then Moment of Inertia requires definition,—the scalar sum of the product of every particle of the body by the square of its distance from an axis.

Thus, in "Matter and Motion," Maxwell reduces the uniplanar motion of any rigid body to an equivalent particle pair, rigidly connected, having the same total weight, the same centre of gravity, and the same moment of inertia about the centre of gravity.

The compound clock pendulum of Huygens is replaced in this way by its equivalent pair of particles, one being placed at O in the axle of suspension, and the other will be at P the centre of oscillation; and then OP is the length of the simple equivalent pendulum, a plumb bob P at the end of a thread OP.

Provided with these additional ideas in dynamics, the young engineer will be able to investigate the motion of the revolving parts of his machinery, such as a flywheel, a revolving shaft, a screw propeller, and the influence of the rotation of the wheels of a carriage.

Whatever the system of units employed, it is essential in the dynamical interpretation for g to be assigned its proper place, here under v . It must not be allowed to straggle and take cover under W , as in the old-fashioned treatise, where the author, to save trouble in writing and printing, adopted the mischievous delusive plan of replacing his W/g by the single letter label M , and then calling it the mass, a quantity *sui generis*, not its *Corpus*. He then wrote, with $v/t=f$, the acceleration, time rate of growth of velocity,

$$(4) F = Mf, \text{ with } W = Mg.$$

These relations are not seen in the "Principia-Elements," where g does not occur explicitly but is concealed in the length L of the seconds-pendulum, $g = \pi^2 L$. The "Principia" is chiefly kinematics; very little of kinetics until the second book, and then of experiments on fluid resistance, always expressed in gravitation units.

Lex III. Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi.

According to Maxwell, this Law—Action and Reaction are equal and opposite—amounts to no more than the definition of a stress, a pull or thrust, tension or pressure.

The sequel of Corollaries of Law III is important in introducing the ideas of vector composition, the conservation of momentum, and immunity of the centre of gravity to the internal actions and stress.

The Law was put forth probably as a contradiction to some accepted law in vogue before Newton, now forgotten. In some recent figurative language of the Press, we read "The pendulum is always swinging. Action, especially if violent, is apt to entail reaction." The former Law can still be traced in the popular idea current that the horse advances by pulling the cart harder than the cart pulls back. I remember a similar question about a double-headed express train; I was asked to explain what would happen if the second engine was going faster than the first: pulling harder I presume was meant.

The definitions come first in the "Principia," but we prefer to discuss them after the Laws, when the ideas they imply have been employed already in some tangible application, and so are capable of a better appreciation, and we can refer to them. Abstract

definition requires to settle on some hard base of fact and comes after action in order of thought.

Definitio I. Quantitas Materiae est mensura ejusdem orta ex illius Densitate et Magnitudine conjunctim.

According to Mach, this is really no more than a definition of density, and *quantitas* is used as a synonym for *Corpus*, *Moles*, *Massa*, *Pondus*, five names to one entity. Nomina-Entia non sunt multiplicanda praeter quam necesse est.

Here *Corpus* would connote a body, *Moles* or *Mola* the bulk, *Massa* the aggregation of its stuff, and *Pondus* the quantity of its stuff measured out in a balance against standard lumps of metal called weights; revealed also roughly in the heft required to lift the body off the earth's surface.

The Greek equivalent for *Massa* would be *μάγμα*, *μάζα*, something kneaded and fashioned into shape; and the distinction in Latin between the words is brought out clearly in Ovid's lines (*A. A.* iii. 219):

"Quae nunc nomen habent operosi signa Myronis,
Pondus iners quondam duraque massa fuit";

assigning the quality of Inertia to *Pondus*.

But because Newton in this definition goes on to say—Innotescit ea (Quantitas) per corporis cujusque Pondus. Nam Ponderi proportionalem esse reperi per experimenta Pendulorum accuratissime instituta, uti posthac docebitur (meaning experiments to prove that the quality of the Matter does not matter)—it is rigidly insisted to-day in elementary instruction that *Pondus* should never be used except in this subsidiary sense of the accident of the gravitation of it, due to its situation on the surface of the earth. Moreover, we find Newton using *Pondus* elsewhere in both of the meanings of ordinary language; as, for example, in his preface—Unde solvitur in omni aptorum instrumentorum genere Problemata. Datum pondus data vi movendi. Sic pondera aequipollent ad movenda brachia Librae, quae oscillante Libra sunt reciproce ut eorum velocitates sursum et deorsum: hoc est, pondera, si recta ascendunt et descendunt, aequipollent, etc.

Definitio II. Quantitas Motus est mensura ejusdem orta ex Velocitate et Quantitate Materiae conjunctim.

This is the quantity called momentum to-day, our Wv , lb.-ft. per sec. And—Motus totius est summa motuum in partibus singulis—requires amplification to-day of "sum" to "vector sum."

Definitio III. Materiae Vis Insita . . . etc., is qualified as undistinguishable from *Inertia massae*, the *pondus iners* of Ovid, so here the two names are convertible, and one of them would serve.

The *Vis Impressa*, *Vis Centripeta*, *Vis centripetae Quantitas Absoluta*, *Acceleratrix*, *Motrix* (felt forcibly on the top of a motibus), and so on of the subsequent Definitions display a curious profusion of the word *Vis*, as much as in Hooke's vaunted Law: Ut Tensio sic Vis.

Vis, like moment, momentum, moment of momentum, moment of inertia, is a word too hackneyed in dynamics. A. N. Whitehead, in the "Concepts of Nature," uses the word moment to mean "all Nature at any instant." "Two moments of the same family are parallel." "A point flash of Nature is an event particle."

In Definition I, density is taken as the primary property of matter, although left undefined by Newton; while *Quantitas Materiae*, our W or P , is the product of density and volume.

The *Materiae Vis Insita* of Definition III is described as the same as *Inertia Massae*. This, however, is not the definition of *Massa*, but *Inertia*, although the two are treated as the same thing in modern interpretation.

Newton is not consistent with himself, as asserted, in always using *Pondus* as meaning the attraction of the earth on a body on the surface. As often as not he uses *Pondus* in the popular acceptance, as in the Act of Parliament, and a search in the "Principia" will reveal numerous instances.

This distinction, insisted on so carefully in modern instruction, was ignored in language and thought till about fifty or sixty years ago, when Absolute Measure was first introduced into dynamical teaching.

The artless definition of Mass, as the quantity of matter in the body, is near enough to serve in a dictionary, as a synonym in one line. It is merely the selection of a new name as a label in the long list already in Def. I. But a real definition will give at the same time the best way to measure the quantity. In a recent Royal Society memoir, on "mass determination" as the author is careful to call it, the question of its measurement turned on a "Study of the Balance, in its greatest precision, in a projective series of weighings of small masses," the most accurate of all physical operations we know.

Libra, sign in the Zodiac of the Balance, is an appropriate emblem of justice holding the scales.

It is contrary to the strict legal language of the Act of Parliament on weights and measures to start off with another artless definition—the weight of a body is the force with which it is attracted by the earth. At that rate, what is the weight of the moon, the sun? The definition is not supposed to apply to a body, so long as it is not terrestrial.

The attraction of the earth on the pound weight as the unit of force (gravitation) will never be abandoned by the engineer, as it is susceptible to the same degree of accuracy of measurement as the operation of weighing.

But when Tait took in hand the reform of dynamical teaching, he altered the equations in our form of (1), (2), (3), (4) in a new way, with the view of exterminating g . He discarded the old *sui generis* mass, with unit of g lb., and taking mass in its new meaning of the invariable quantity of matter in the body, he measured it in terms of the Act of Parliament unit of weight, the pound weight. This involved him in a change in the unit of force, to what was called a poundal, such that the engineer's gravitation unit of force, the pound (force), was equivalent to g poundals.

Tait's change merely amounted to labelling M the quantity formerly labelled W . But he insisted on retaining $W = Mg$, and so measuring what he called weight in poundals, contrary to the strict law of the Act, and rendering himself liable to a fine for every offence. Better if Tait had retained the letter W for lb., writing the equation $P = Wf$, and rejecting the useless $W = Mg$, as perpetuating the old *sui generis*, and breaking the law in the Act of Parliament.

This trouble of mere terminology would be excoriated if the habit was inculcated of always stating the unit of a dynamical quantity, as, for example, of a mass M , g , a weight W , lb. The engineer refuses to accept the poundal or to give a weight W in poundals. Scrap the name as useless, except for passing certain examinations.

To the masses in general the word mass implies a combination of bulk and density as in Definition I, as when we speak of mass of stuff, the mass of the earth—"Die Erde und ihre eigene ungeheure Last" (Mach). In ordinary language the mass will mean the multitude, or majority, as in the statement attributed to Herbert Spencer, "The mass of woman is insensible to gravity," which might mean a reminiscence of the ballroom floor; but this was before the women began to take themselves so seriously; and when we read the critic's snarl of the "Vast Mass of his writings consigned to Oblivion," Vast Mass here is forcible—feeble for *Major Pars*.

The word is spelt *Maas* in German; "Mass für Mass" is the title of the German version of Shakespeare's play "Measure for Measure."

Wegener's Hypothesis of Continental Drift.¹

By PHILIP LAKE.

WEGENER'S hypothesis is based on the idea that the continental masses are patches of lighter rock floating and moving in a layer of denser rock, and this denser rock forms the floor of the oceans. Following, with a slight alteration, the terminology of Suess he calls the lighter material the Sial and the denser layer the Sima. Suess uses the words Sal and Sima, and thinks that the Sal covers the globe completely.

I shall not here discuss the possibility of Wegener's conception. He does not profess to explain completely why the continents should move, but he claims to have proved conclusively that such movement has taken place. It is the evidence on which he relies, and more particularly the geological evidence, that I propose to examine.

One of the arguments on which he lays great stress is derived from the relative frequency of different heights and depths upon the earth. His diagram of frequencies shows two well-marked maxima, one at about 100 metres above sea-level and the other about 4700 metres below it. Wegener concludes that two distinct surfaces standing at these two altitudes must have been involved in the subsequent movements. He assumes that these surfaces were originally level—or, more strictly, equipotential—and that they were the surfaces of the Sal and the Sima respectively. He holds that if originally there were only one such level, the deformation of that level could not produce two maxima and "the frequency must be regulated according to Gauss's law of errors."

In reality, if it is only a single level that has been deformed, it is improbable that the resulting altitudes

¹ Abridged from an address to the Royal Geographical Society on January 22.