

Gradient. Degrees Centigrade per metre.	σ in miles.		k (Indian).	
	$t=0^\circ$	$t=10^\circ$	$t=0^\circ$	$t=10^\circ$
0.000 (isothermal)	16,980	18,220	0.117	0.109
0.006 (average)	20,600	22,100	0.096	0.090
0.010 (adiabatic)	24,000	25,760	0.082	0.077
0.03414	infinite		zero	

Mr. Mallock's value, 14,900 geographical miles = 17,150 miles, agrees nearly with the isothermal value for $t=0^\circ$. (In my former letter I had not recognised that Mr. Mallock's result was in nautical miles.) The result is too small as a usual value, because he takes the temperature gradient as zero and the surface temperature to be freezing point. Dr. Ball's explanation is incorrect as pointed out by Commander Baker (January 26); and further in that he states in his second paragraph that the difficulty is not to be got over by any consideration of temperature gradient.

Commander Baker, in his letter (January 5) has arrived at a similar result, for a horizontal ray, as I have. The temperature gradient, however, of 1° C. per 200 feet, which he says will give my results, is in error; it should be per 600 feet.

In the second paragraph of this letter, Commander Baker says that neither Mr. Mallock nor I give an adequate presentation of the facts, in that the assumption is made that the ray is circular. I do not think that this deduction can be made rightly from my former letter of August 11. I may say at once that I entirely agree that the ray is not in general circular, especially when the ray is close to the earth or sea surface. However, in cases met with in land surveys (excepting rays which continue very close to the ground) one may compute the refraction practically by the use of a coefficient of refraction which represents the curvature at height $(2h_a + h_b)/3$, as stated in my letter. The use of different coefficients of refraction for different heights essentially involves the idea of a ray of varying curvature except in the case of a truly horizontal ray.

Now work on the diurnal change of refraction on inclined rays shows up the importance of the varying conditions of temperature gradient in the layers near the earth. I have not yet been able to reduce the case of rays, which lie mostly or largely in these lower layers, to a formula, though I think there is fair hope of doing so in some cases. Extreme cases, in which there is obvious and varying mirage, will not be amenable to treatment: but I think a ray, 20 feet above the surface, probably will. But I gather that Commander Baker is chiefly interested in rays over the sea, at a height of 30 feet or less. In the Survey of India such cases naturally do not arise, and I have not had any observations of this kind to consider. However, in my Professional Paper No. 14 (Survey of India) I have given some deductions as regards dip of the horizon (*vide* pp. 96-100), arriving at the formula

Dip in seconds from point at height $h =$

$$56.33(h' - 15.13\Delta t)^\frac{1}{2}$$

where $\Delta t' = F\Delta t$, $h' = h(1 - 0.2204F)/0.7796$, $F = 519.4/t$, $t + \Delta t$ and t being the absolute temperatures at levels of observer and sea respectively. This formula is based on $\cos(\text{dip}) = (1 + h/r)^{-1} \mu_0/\mu$ which involves only the terminal values of μ .

I have tabulated the corresponding dip in Tab. LIII. *loc. cit.* for various values of h' and $\Delta t'$, and I should be very interested to hear from Commander Baker or others to what extent my formula represents the facts of observations. J. DE GRAAFF HUNTER.

Survey of India, Dehra Dun, U.P., India, March 2.

I AGREE with Dr. Hunter that my letter of January 5 contained a numerical error when I stated that a temperature gradient of 1° C. per 200 feet would give a ray curvature corresponding to the refraction coefficient given in his letter of August 11, 1921.

Dr. Hunter also takes me to task for my comment that both he and Mr. Mallock assume the refracted ray to be circular. I think I have a certain amount of justification for this, as in his letter of August 11 he speaks of the curvature of the ray "tacitly assumed to be circular," although later it is true that he states that the coefficient of refraction has different values at different heights.

It was rather in connection with the formulæ upon which the nautical tables for the dip of the sea horizon are based that I take exception to any assumption that adequate results can be obtained unless variations of curvature are considered.

As stated in my letter of January 5, it is impossible to draw a circle which touches the surface of the sea and also becomes horizontal at a height of say 30 feet above the sea, and unless consideration is given to a form of ray path that can satisfy these conditions it is impossible to get a zero value for the dip.

Dr. Hunter quotes from his Professional Paper No. 14 (Survey of India) a formula which he has set out there from which the dip is to be evaluated, and asks to what extent this formula represents the facts of observations. I have, unfortunately, no data of measurement of the dip made in connection with the temperatures of the sea level and at the bridge, but on theoretical grounds I cannot admit that this formula is correct. It will be seen that the dip becomes zero whenever $h' = 15.13\Delta t'$, which is equivalent to saying that, if the temperature rises uniformly 1° F. per 15 feet, the dip is zero at all heights. Consider now what will happen to a ray of light which starts off from the surface of the sea tangentially. In an atmosphere of uniform refractive index that ray would proceed in a straight line and ultimately depart from the earth entirely. With a refractive index that diminishes with height the ray will be bent towards the earth, and if the rate of diminution is great enough that ray will at some point become horizontal and the dip will be zero. Let us say that this point is at a height of 30 feet above the sea. Dr. Hunter's formula requires that the temperature should be 2° more at 30 feet than at sea level, and if the rise of temperature is uniform in this 30 feet his formula also requires that the dip should be zero, and therefore the ray horizontal, at all heights below 30 feet. This is obviously a fallacy, for if the ray was always horizontal it would never reach the height of 30 feet at all.

The fact is that in an atmosphere where the layers of uniform refractive index are spheres concentric with the earth, the dip can only be zero, if at all, at one height. Below that height the dip will be positive with a maximum value at some lower level; above that height no ray tangential to the earth's surface can be seen at all, and the depression or elevation of the sea horizon requires an entirely different explanation.

The equation upon which Dr. Hunter's formula is based brings out this point quite clearly. This equation is $\text{Cos}(\text{dip}) = \mu_0 r / \mu(r + h)$.

In an atmosphere in which $\mu(r + h)$ is, at some height, less than $\mu_0 r$ the dip becomes imaginary, for its cosine is greater than unity. The dip could only be zero for all heights for an atmosphere in which $\mu(r + h)$ is constant, and in such case a ray once horizontal would remain horizontal for a complete circuit of the earth.

THOS. Y. BAKER.

Admiralty Research Laboratory,
Teddington, Middlesex, April 6.