Stellar Magnitudes and their Determination.¹

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II.—APPARENT MAGNITUDES: (b) PHOTOGRAPHIC.

WITH the application of photography to astronomy it was inevitable that attempts should be made to determine apparent magnitudes by photography. Visual observations are slow, for every star must be compared individually, and the telescope reset for each. Photography effects a great economy in observing time at the telescope, for when a plate is secured its measurement may be undertaken at any convenient time. The photographic plate, however, is sensitive to a different region of the spectrum from the human eye; if a blue and a red star appear of equal brightness to the eye, the former will be recorded as much the brighter by the photographic plate. The photographic and visual scales of magnitude will therefore not agree with one another. The difference, photographic *minus* visual magnitude, for any star is called the "colour-index" of that star, providing as it does a measure of the colour of the star; the redder the star, the larger is its colour-index.

The determination of photographic magnitudes is based upon the two following conventions: (i) the light ratio shall be the same as that adopted for visual magnitudes, its logarithm being, therefore, 0.40; (ii) for stars the spectra of which are of the type A₀ in the Harvard classification (*i.e.* in which the most conspicuous feature is the Balmer series of hydrogen lines), the photographic and visual magnitudes shall be equal. If this holds for stars of, say, the 6th magnitude, it will hold also for stars of all magnitudes, by (i). Stars which are bluer than type A₀ have small negative colour-indices; those which are redder have positive colour-indices, the values for the reddest stars being larger than two magnitudes.

The accurate determination of photographic magnitudes is a problem which is much more complicated than it appears upon the surface, and beset with many difficulties. It consists essentially of two distinct problems: the absolute determination of the magnitudes of a suitably chosen series of stars, and the extension of this series to determine the magnitudes of other stars by comparative methods. Although much work has been done at Harvard, Mount Wilson, Greenwich, and elsewhere, there remain discordances which require further investigation before photographic photometry can be regarded as having been placed upon a definite and satisfactory basis.

The area around the North Pole has been chosen in the northern hemisphere as the most suitable area for the absolute determinations, as it is always available for use for comparative methods. A sequence of stars has been chosen by the Harvard observers, called the "north polar

sequence," which are graded in magnitude so as to provide the necessary basis for comparison, and the magnitudes of these stars have been carefully determined by the use of various methods. The difficulty of the absolute determination of these magnitudes is increased by the complication introduced by the law of photographic action. It has been found that, for a given light intensity, I , the photographic effect produced does not increase uniformly with the time, so that the same photographic effect is not obtained by, say, doubling the intensity and halving the time of exposure. In fact, the relationship between the intensity and the time of exposure required to produce a given photographic effect is of the nature $I^q t = a$ constant, where q is a constant for any given type of plate, but has different values for different types, although averaging somewhat about 0.8. Now most of the methods of determining absolute photographic magnitudes depend upon successive exposures given on the same plate, some means being employed to reduce the intensities during one of the exposures. It is clear that, for all photometric work, the times of the two series of exposures must be exactly equal, and then the comparison of the images obtained from the two exposures only involves the assumption that the intensities which in equal times produce equal photographic effects must be equal.

If, then, the photographic effects produced by a series of stars in the first exposure are denoted by:—

$$i_1, i_2, \dots, i_s, i_{s+1}, \dots, i_n,$$

and by the same stars in an equal exposure, in which the brightness has been reduced in a proportion equal to a difference of Δm in magnitudes, are:—

$$i'_1, i'_2, \dots, i'_r, i'_{r+1}, \dots, i'_m,$$

then, if $i_s = i'_r$, it follows that the magnitudes of stars r and s differ by Δm . In this way, differences of magnitude are determined, as in the case of visual observations with a photometer. The zero of the magnitudes must be chosen in accordance with the convention referred to above.

In practice, of course, it rarely happens that two stars can be found the photographic intensities of which in the two cases are exactly equal. The procedure usually adopted is to estimate the photographic effects against an arbitrary scale, and then to use the known fact that the two images of any one star correspond to a magnitude difference, Δm , in order to determine the values of the scale intervals. The magnitude of every star can then be read off.

Various devices have been used to reduce the intensities by a known amount. One method, which has been extensively used at Greenwich, is

¹ Continued from p. 146.

to place over the object glass of the telescope a coarse grating of parallel wires; from the dimensions of the grating the magnitude difference between the principal and first diffracted image can be calculated. Thus one of the Greenwich gratings, illustrated in Fig. 3, which has wires of diameter 1.72 mm., and a total grating interval of 7.0 mm., produces a magnitude difference of 2.66 m . An enlargement of a portion of a photograph obtained by this grating is shown in Fig. 4.² It will be seen that the first diffracted images are round, but that the second diffracted images are elongated by dispersion, and not suitable for comparison. The use of the grating has the advantage that all the information required can be obtained from one exposure, the principal and diffracted images corresponding to two series of images differing by a known magnitude. Any possibility of

FIG. 3.—Diffraction grating used in stellar photometry.

error, which might otherwise be introduced owing to a change in atmospheric conditions between the exposures, is thus avoided. Other methods which have been employed, principally at Harvard and Mount Wilson, consist in using wire-gauze screens, or rotating sectors, the reduction in luminosity being measured by a photometer in the laboratory in the first case, and calculated in the second case, or in the reduction of the aperture by circular diaphragms. Although the latter method changes the diffraction pattern of the images, no disturbing effects seem thereby to be produced; it is, however, objectionable in the case of a refractor, as the light passes through different parts of the object glass in the two cases, and the difference in absorption introduces errors. Other methods have been employed, but less frequently than those just referred to. It is not convenient to reduce the magnitude too much at one step, as errors are liable to arise. A reduction of

² On account of the difficulty of reproducing satisfactorily faint star images, Figs. 4 and 5 are not actual reproductions of photographs, but are drawn from the photographs. For this reason some of the images do not appear as true discs.

about 5 m is a practicable limit. If stars of a wide range of brightness need to be compared, it is preferable to make the comparison by two steps.

The diameters of the star images increase with the length of exposure. The images are compared with a scale obtained by giving exposures, preferably with the same instrument, on a real or artificial star, the length of the exposures being so graduated that the difference in magnitude between consecutive images is very nearly constant. The sizes of the star images are compared with those of the scale, interpolation to tenths being made between the scale images. The comparison becomes difficult for very bright or very faint stars, so that it is customary to measure only those images which come within a certain interval of the scale; for the brighter and fainter

FIG. 4.—Portion of photograph obtained with diffraction grating.

stars, shorter and longer exposures respectively must be given. Corrections have to be determined and applied for the distance of the star image from the centre of the plate, and for atmospheric absorption, the latter as in the case of visual photometry. The former correction arises from the curvature of the field; if focussed exactly at the centre, the edges of the plate are not quite in focus, so affecting the size of the images. It is convenient to make the focus come somewhere between the centre and the edge of the plate.

In this way the magnitudes of the stars comprising the north polar sequence have been determined. There is a good accordance between the several determinations of the magnitudes in the range 10 m –15 m , but in spite of the extensive investigations which have been made, there remain systematic differences between the magnitudes obtained for the brighter stars at Mount Wilson and Harvard which exceed 0.25 m , and this discordance illustrates how much more

difficult is the absolute determination of apparent magnitudes than might be gathered from the above brief account of the theory.

The magnitudes of stars in other areas are based upon those of the north polar sequence. The procedure involves photographing the area in question and the pole area upon the same plate, giving the same exposures, and then comparing the two sets of magnitudes against an arbitrary scale, using the known magnitudes of the pole stars to standardise the scale. Actually, it is customary to expose on the pole, then to give two exposures on the field, followed by another exposure on the pole. In this way the effect of any uniformly progressive change in the sky during the time occupied in taking the plate is eliminated. A portion of a photograph showing a comparison of the polar area with another area is reproduced in Fig. 5. In this figure the pole stars can be distinguished by the fact that the displacement between the two images is not parallel

FIG. 5.—Portion of photometry plate showing pole and field stars.

to a *réseau* line. An alternative method of measurement is to measure the diameters of the star images in a micrometer. For a wide range of magnitude, the relationship $m = a + b\sqrt{d}$, in which a , b are constants, and d is the diameter of the image of a star of magnitude m , is found to hold. The constants a and b can be determined by a least squares solution, using the data obtained from the stars of known magnitude.

A slightly different method of procedure is to take photographs at some distance out of focus, the plate meanwhile being given slight periodic motions in two perpendicular directions by means of a device invented by Schwarzschild, and called by him a "Schraffierkassette." The central portion of the image so obtained is uniformly grey; the plate is measured in a comparator, the central portion of the image being seen surrounded by a grey field, the density of which can be varied, produced by a plate with a uniformly graduated density. The position of the latter is varied until the tint of the star image matches that of the surrounding field. The readings can be standardised

by stars of known magnitude. This method possesses the advantage that it is not appreciably affected by bad definition, whereas when images in focus are being dealt with, bad definition causes woolly edges, and the images are not then exactly comparable with those of the scale.

Either a reflecting or a refracting telescope may be employed for the determination of photographic magnitudes. Owing to the absorption produced by the object glass of a refractor, there is a slight relative difference, depending upon the colour of the star, between the magnitudes obtained by the two types of instrument. The differences in magnitude can be expressed as a linear function of the colour-index, and the constants of the relationship require to be determined for each instrument from a comparison of the results obtained from white and red stars. The phenomenon becomes of some importance when the scale of magnitudes is extended to faint stars, for it has been shown by Seares that the faint stars are, on the average, considerably redder than the brighter stars. The effect of this will be to give systematic errors in the case of a refractor equivalent to the errors that would be introduced by the use of an incorrect light ratio.

It is of interest to compare the numbers of the brightest stars down to a limiting magnitude of 7.0m in the case of visual magnitudes with the corresponding numbers in the case of photographic magnitudes. The visual estimates were made at Harvard, the photographic at Greenwich.

Total Number of Stars to Various Limits of Magnitude.

Brighter than	Visual	Photographic
1.0	11	11
2.0	38	41
3.0	111	138
4.0	300	454
5.0	950	1,480
6.0	3,150	4,750
7.0	9,810	14,960

The brightest star, both visually and photographically, is Sirius; its spectrum is of type Ao, so that both visual and photographic magnitudes are $-1.6m$.

The economy in observing time effected by the application of photography to the determination of magnitudes has resulted in visual magnitudes being determined by photographic methods. This is effected by the use of isochromatic plates in conjunction with a yellow filter, which is found by experiment to give a spectral-intensity curve similar to that of the normal human eye. This can be tested by means of the magnitudes determined visually with a photometer, and the visual-scale can then be continued to magnitudes much fainter than those which have been determined visually. For distinction, it is customary to call the magnitudes so determined "photo-visual" magnitudes; photo-visual magnitudes of the stars of the north polar sequence have been determined at Mount Wilson down to a limit of 18m, much

fainter than would ever have been possible by ordinary visual methods.

The photographic and visual or photo-visual magnitudes of a star having been observed, the "colour-index" is at once obtained. There is a marked dependence of the colour-index upon the spectral type of the star. The basis of the classification of the spectra of stars adopted at Harvard, and now universally accepted, was entirely independent of magnitude or colour considerations, and depended solely upon the type of spectrum. The spectra of the types B, A, F, G, K, M were found to show in this order a progressive change from bright-line to absorption spectra, and the order is intimately bound up with the problem of stellar evolution, and also with the temperature of the stars. The colour-indices found in three separate investigations for stars with spectra of different types are given in the table, together with the temperature of the stars, derived by Russell on the hypothesis that the stars radiate as black bodies. In accordance with the convention on which photographic magnitudes are based,

the colour-index for type A₀ is zero in each investigation.

Colour-Index.

Spectrum	King	Parkhurst	Schwarzschild	Temperature
B ₀	-0.32	—	—	20,000
B ₅	-0.17	-0.21	-0.20	14,000
A ₀	0.00	0.00	0.00	11,000
A ₅	+0.19	+0.23	+0.20	9,000
F ₀	0.30	0.43	0.40	7,500
F ₅	0.52	0.65	0.60	6,000
G ₀	0.71	0.86	0.84	5,000
G ₅	0.90	1.07	1.10	4,500
K ₀	1.16	1.30	1.35	4,200
K ₅	1.62	1.51	1.80	3,200
M	1.62	1.68	—	3,100
N	—	2.50	—	2,300

It will be seen that the colour-index increases almost uniformly from class to class, and that when either the photographic or visual magnitude, and either the colour-index or the spectral type, are given, it is possible to determine the remaining data with very little uncertainty.

(To be continued.)

Ocean Tides.

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THE tides of the oceans form the most magnificent dynamical phenomenon of our planet, and yet we are extremely ignorant of even their main characteristics. It is only in the immediate neighbourhood of land that they become directly observable, and it is practically only here that they have hitherto been observed.

Much has been done in the way of recording coastal tides and in analysing the records obtained, yet very much more remains to be done even for the purpose of preparing accurate commercial predictions. In this connection the most urgent need is the study of the meteorological effects. Owing to these effects, the tide in a harbour on any day may be several feet different from that due to astronomical causes, which alone appears in the tables of predictions. Now this is of the very gravest concern to harbour authorities, for, in docking a large vessel, to get less water than was expected may be very serious, while to refrain needlessly from docking through fear of this possibility is a fruitful source of delay and expense. And this is merely an instance.

The up-and-down motion of the water-surface is accompanied by oscillating currents. Much rough information is in existence concerning the nature of these currents near land, having been gathered chiefly by naval authorities, as it is of the utmost importance in navigation. But the number of places at which accurate observations of currents have been made with modern instruments is extremely small. No such observations are on record, for example, for the Irish Sea. When the problem of predicting the meteorological effects comes to be tackled in a way likely to lead to success, these shallow-water currents, which

are mainly instrumental in producing the local wind effects, will require thorough observation.

But when the tides are viewed scientifically as the oscillations of a great dynamical system, these coastal tides, that almost alone have been observed, appear as the mere fringe, so to speak, of the essential phenomenon. It is in the vast bodies of water constituting the great oceans that the tides have their real being, and the coastal tides themselves will never be completely understood until we know the great oceanic tidal movements. The meteorological disturbances may arise wherever the tides arise, and we want to know, for instance, what effect certain meteorological conditions over the Atlantic will have on the tides in our harbours.

On the side of pure science many problems of wide geodynamical and cosmical interest require as data a knowledge of the ocean tides.

Now it is believed that not a single accurate observation of either tidal elevation or tidal current has ever been made in the deep water of any of the oceans. The best knowledge we possess of mid-ocean tides consists in observations on the shores of oceanic islands, and even this knowledge is not nearly so complete as we could wish.

Mathematically, the tides are "determined" by the size and shape of the ocean basins and certain astronomical data, but the complete solution of the problem is not within the sight of the present generation of mathematicians. If all the possible free oscillations of the oceans could be discovered, then the actual tides could be calculated with ease by a principle which is a generalisation of that of resonance.

Various guesses have been made as to the