

not those usually accepted, nor will Fru Wiig of Bossekop feel happy in appearing as "Mrs. Wiggs." "Gastivare" (p. 125) is neither Finnish nor Swedish, and "kestikievari" would seem to be the word intended. Mr. Butler, however, can drive reindeer, just as he can follow game in Africa, and the main thing is that he accommodates himself so skilfully to his surroundings. Even if we cannot hope to follow him, and may be content to view the wilds of Lapland from Abisko or the top of Kiirunavaara, we feel

statistical mechanics and their applications to the problems of stellar dynamics. Since the positions and motions of individual stars are known only in a few instances, it is impossible to treat the motions of stars by the ordinary methods of classical mechanics, so that statistical methods have to be adopted. Important investigations in stellar dynamics have been made recently on this basis by several investigators, more particularly by Eddington and Jeans. There are two fundamentally different methods of treatment: (a) The



FIG. 2.—Lapp tent and sledge at Jukaskjärvi. From "Through Lapland."

[Photo F. H. Butler.]

something, as we turn his pages, of the dry, healthy air and the crispness of the arctic snow.

GRENVILLE A. J. COLE.

STELLAR DYNAMICS AND STATISTICAL MECHANICS.¹

THE five papers referred to below do not form a logical sequence of discussion, but are related to one another in that they are all more or less directly concerned with the methods of

¹ (1) "Statistical Mechanics, based on the Law of Newton," *Lund Meddelande*, Ser. ii., No. 16. (2) "Ueber den Satz von den Gleichen Verteilung der Energie," *Lund Medd.*, Ser. i., No. 79; *Arkiv för Mat. Astr. och Fysik*, Bd. xii., No. 18. (3) "Ueber hydrodynamisches Gleichgewicht in Sternsystemen," *Lund Medd.*, Ser. i., No. 82; *Arkiv för Mat.*, etc., Bd. xii., No. 21. (4) "Conceptions Monistique et Dualistique de l'Univers Stellaire," *Lund Medd.*, Ser. i., No. 81; *Scientia*, vol. xxii., p. 77 (1917). (5) "Eine Studie über die Analyse der Sternbewegungen," *Lund Medd.*, Ser. i., No. 78; *Arkiv för Mat.*, etc., Bd. xii., No. 10. All by Prof. C. V. L. Charlier.

stars may be compared with the molecules of a gas, and the effect of the various encounters considered, the discussion proceeding along the lines of gas theory. (b) It may be supposed that the encounters of stars have but small effect, so that the stars may be regarded as describing orbits under the general attraction of the stellar system as a whole, the discussion then proceeding along the lines of hydrodynamics. Both methods may be expected to give results of value for the general theory.

Prof. Charlier has adopted the first of these two methods in (i), and has worked out a kinetic theory for the stars based upon Newton's inverse square law of attraction; in gas theory the treatment has usually supposed either that the molecules are elastic spheres or that they repel each other inversely as the fifth power of the distance.

The latter law is artificial, but was used by Maxwell because it introduced considerable simplification into the discussion. Where stars are concerned it is necessary to distinguish between real collisions and encounters. The latter occur when two stars approach one another sufficiently closely to produce a relative change in path without actually colliding. The number of collisions will naturally be considerably less than that of the encounters. The fundamental general equation of statistical mechanics is formed, and the effect of the collisions and encounters obtained. The discussion follows closely along normal lines. The integration of the fundamental equation when the solution is a frequency-function of type A is performed, the solution being rather more complicated than for Maxwell's law of repulsion. The time of relaxation, which is a measure of the time taken by the system to reach a steady state, is found to be about 10^{16} years. Jeans had previously obtained, by somewhat different reasoning, a value of 10^{14} years, which is of the same order of magnitude.

In (2) some of the results obtained in (1) are applied to prove the law of equipartition of energy for the stars. The proof is elementary and applies only for translational velocities, any possible energy of rotation not being taken into account. As regards translational energy, recent results indicate that the most massive stars have the slowest velocities on the average, and *vice versa*, which is in the sense required by equipartition. But whether there is anything like real equipartition, even for translational velocities, we do not know; still less do we know to what extent the energy of rotation shares in the equipartition. In any case, we should not expect equipartition to hold unless the system had practically reached a steady state, and other evidence must be adduced to settle this point.

In (3) the hydrodynamical analogy is used, the average motion of a small group of stars under the general attraction of the stellar system being considered, neglecting the effects of encounters and collisions on the motion of individual stars. The equation of motion for a steady state is derived from (1) and integrated. The result is obtained that in a star cluster, in which the stars are symmetrically distributed about an axis, in which there is hydrodynamical equilibrium and ellipsoidal velocity surfaces, these surfaces must be spheroids with their axes of rotation perpendicular to the radius vector from the centre of the cluster. The same result had previously been obtained otherwise by Jeans. It was proved by Schwarzschild that the velocity surfaces are approximately spheroids with their rotation axes directed towards the *vertex*. Jeans, through insufficient evidence, had concluded that this direction was not perpendicular to the radius vector. On the other hand, Prof. Charlier, on the evidence afforded by recent investigations at Lund, concludes that the two directions are perpendicular. Jeans has since accepted the evidence on which Prof. Charlier bases this conclusion. The result supports, but

does not prove, the supposition that our stellar system is in such equilibrium, for there are other factors to be taken into consideration.

In (4) Prof. Charlier discusses and compares what he calls the monistic and dualistic conceptions of the stellar universe. According to the former, the universe can be considered as a single system which, if it has not actually attained a steady state, is on the way to doing so. By the latter he means the hypothesis that there are two intermingling star-streams, though it is doubtful whether the originators of that hypothesis ever conceived that there were two streams of stars approaching and passing through one another. Our knowledge of stellar motions is derived almost entirely from the nearer stars, and it would be dangerous to make so sweeping an assertion. Reasons are advanced by Prof. Charlier for supposing that the methods of statistical mechanics as developed in (1) can be applied to the monistic conception, and an endeavour is made to show that the state of motion in our system is comparable with the results given by the kinetic theory. The time of relaxation obtained in (1) was thought by Jeans to be too long for our system to be considered as yet in a steady state. Prof. Charlier brings forward evidence to show that the velocities of the stars are in qualitative agreement with the requirements of the kinetic theory [see (2)], and that red stars are more nearly in statistical equilibrium than the younger blue stars. The results obtained in (3) also supported the idea of a steady state. To Eddington's difficulty of believing that the evidence of scattered clusters of stars moving with a common velocity, such as the Ursa Major cluster, can be explained if the chance attractions of stars passing in the vicinity have an appreciable effect on stellar motions, Prof. Charlier replies that it is possible that such clusters are but the remnants of much larger clusters, most of the members of which have succumbed to encounters with other stars by the way. The sparseness of the stars in these clusters may be held to support this view. Furthermore, Jeans has shown that a compact globular cluster moving through another mass of stars will be spread out into a disc-like arrangement, perpendicular to the direction of motion. The conditions of Jeans's discussion cannot be exactly reproduced in the stellar universe, but it is interesting to note that Turner has shown that the Ursa Major system has approximately this shape.

The fifth paper is a valuable discussion of the various methods which have been used for analysing stellar motions, and forms a convenient summary for purposes of reference. The analysis on the simple hypothesis of a single star-stream, on that of two star-streams developed by Kapteyn and Eddington, on the ellipsoidal hypothesis of Schwarzschild—all of which are based upon the directions of the motions only—and that on the correlation methods developed by Prof. Charlier himself—in which both the magnitude and direction of the motions are taken into account—are discussed and illustrated by application to one particular region of the sky. H. S. JONES.