factory engine-room, at a speed increased to 195 revolutions per minute. It developed frequently 50 brake horse-power with coal gas for several hours together. Since then the engine has been taken to Cambridge, and is now engaged in regular service with a suction-producer, driving the workshops, and producing electric current for the engineering laboratory. It is left to itself like an ordinary gas-engine, giving no trouble at all, and has been in regular work for two years, the total time of running being 5000 hours.

Judging from the success which has so far been obtained, it seems likely that Prof. Hopkinson's method of cooling the cylinder will revolutionise the design and construction of large gas-engine cylinders.

RECENT PAPERS ON VERTEBRATE PALÆONTOLOGY.

A VERY remarkable announcement is made by Mr. J. W. Gidley in vol. lx., No. 27, of the Smithsonian Miscellaneous Collections, namely that an associated series of five upper cheek-teeth of a large ruminant from a Pleistocene cave-deposit near Cumberland, Maryland, U.S.A., indicate an antelope apparently closely related to the elands of Africa. So near, indeed, is the resemblance that the author deems himself justified in referring the fossil to the existing genus, under the name of *Taurotragus americanus*; and the plate showing these teeth alongside those of the existing *T. oryx* goes a long way in confirming his conclusion. It should have been mentioned that the present writer (see Cat. Siwalik Vert. Ind. Mus., part i., p. 1885) has provisionally referred certain teeth from the Indian Siwaliks to Taurotragus (=Oreas); and if the identification be correct, it would explain how eland might have reached America from Asia by the Bering Sea route. Mr. Gidley quotes the occurrence in the Pleistocene of Nevada of remains of certain ruminants described as Ilingoceros and Sphenophalus as corroborative evidence of the former existence of tragelaphine antelopes in America; but he omits to mention that although these genera were at first assigned to that group, they have been subsequently regarded as akin to the American family Antilocapridæ (Merriam, Bull. Dept. Geol. Univ. California, vol. vi., p. 292). If this be correct, is it quite impossible that the supposed eland represents another member of the same group?

In a second communication (*op cit.*, No. 26) Mr. Gidley records the occurrence of a toe-bone of a camel in a superficial deposit at the mouth of Old Crow River, in the Yukon Territory, in association with remains of mammoth, horse, and bison. The occurrence of the camel-bone confirms "the theory of the existence of a wide Asiatic-Alaskan land connection of comparatively recent date, which for a very considerable length of time served as a great highway for the free transmission of mammals between America and the Old World."

As being only in part palæontological, brief notice must suffice for a paper, by Mr. K. S. Bardenfleth, on the form of the carnassial tooth in Carnivora, published in *Vidensk. Meddel. Dansk. naturh. Foren*, vol. lxv., pp. 67–111. After reviewing the various theories of the homology of tooth-cusps, the author proceeds to observe that in order to demonstrate that the simple reptilian tooth-cone is represented by the middle one of the three longitudinally arranged cusps of the Purbeck Triconodon, and that the tritubercular crown has been formed by rotation of the other two, indisputable evidence has yet to be furnished, "first, of the Triconodon-like forms being the ancestors of Dryolestes, &c.; second, of the supposed protocone and protoconid of these being really homologous with the median cusp of Triconodon. One can scarcely imagine how such a rotation could take place, and if Gidley is right in his interpretation of the molar cusps of Dryolestes, the rotation has not taken place, but the so-called protocone is a secondary acquirement; the true protocone is still to be sought in the central one of the three outer cusps. If this holds good the whole nomenclature and theory of Osborn falls to the ground; neither protocone nor protoconid are then identical with the reptilian cone."

Three papers, by Dr. R. Broom, form part 6 of vol. vii. of the Annals of the South African Museum, and relate to the extinct reptiles of the same country. In the first of the triad the author shows that while in Pariasaurus the digital formula is 2.3.3.4.3, in the allied Propappus it is probably 2.3.4.5.3. In the second he describes, as *Noteosaurus africanus*, a new genus allied to Mesosaurus, of which three of the known species are South African, while the fourth is Brazilian. The last paper comprises a classified list of the early Mesozoic reptiles of South Africa, which, apart from dinosaurs, crocodiles, rhynchocephalians, &c., are arranged in no fewer than nine ordinal groups, brigaded in three "superorders." R. L.

AN ALGEBRA FOR PHYSICISTS.[1]

THE principal novelties in Dr. Macfarlane's calculus are that a distinction is made between linear and cyclic successions of vectors, and that the commutative law of addition, as well as that of multiplication, is abandoned. To express what most vectorists write $\beta+\alpha=\alpha+\beta$, Dr. Macfarlane writes $\Sigma(\beta+\alpha)=\Sigma(\alpha+\beta)$. Thus $\alpha+\beta-\alpha$ is not the same as β, but either three sides of a parallelogram, or three coinitial vectors, according as we take linear or cyclical succession. By introducing some subsidiary and rather artificial rules, the author is able to get formulæ that are, in appearance, analogous to the binomial and exponential theorems, and so on.

The actual divergence from quaternion results is not very great, as may be easily shown by an example. Let x be a scalar, α a unit vector, and let $\exp(x\alpha)$ be defined to mean $\Sigma(x\alpha)^n/n!$. Then $\exp(x\alpha)=\cos x+\alpha\sin x$, and if y is another scalar, $\exp(x\alpha).\exp(y\alpha)=\exp(y\alpha)\exp(x\alpha)=\exp\{(x+y)\alpha\}=\cos(x+y)+\alpha\sin(x+y)$.

But, if β is another unit vector,

$$\exp(x\alpha)\exp(y\beta)=\cos x\cos y+\alpha\sin x\cos y+\beta\cos x\sin y+\alpha\beta\sin x\sin y,$$

which differs from $\exp(y\beta).\exp(x\alpha)$, while both, in general, differ from $\exp(x\alpha+y\beta)$: the latter, observe, being by definition the same as $\exp(y\beta+x\alpha)$. Dr. Macfarlane, after writing down his exponential formula, breaks it up into four parts, practically the same as the four given by the quaternion formula above, when written in the form—

$$\exp(x\alpha)\exp(y\beta)=(\cos x\cos y+\sin x\sin y\,S\alpha\beta)+\alpha\sin x\cos y+\beta\cos x\sin y+V\alpha\beta.\sin x\sin y.$$

It must be left to physicists themselves to decide whether Dr. Macfarlane's new algebra is superior to those already available; the need of a sign to express a resultant is a rather severe handicap. To the pure analyst it presents the appearance of a conglomerate, though possibly, with a change of notation, it could be fitted into a place in the family of linear associative algebras. One thing ought to be said: it is not, properly speaking, an "extension" of quaternions. Analytically, the calculus of quaternions is a linear algebra of a perfectly definite type,

[1] (1) "Account of Researches in the Algebra of Physics," I.–III. (Reprint from Journ. Wash. Ac. of Sc., 1912.)
(2) "On Vector-analysis as Generalised Algebra" (Intern. Congress of Mathematicians, 1912.) By Dr. A. Macfarlane.

just as an oak is a perfectly definite type of a tree. Taking $q=x+yi+zj+wk$ as the type of a quaternion, we may generalise the "scalars" x, y, z, w, by making them ordinary complex numbers, or elements of some other algebra, commutative with i, j, k, and combining according to laws of their own. We thus embed the quaternion algebra, so to speak, in a larger composite algebra; but it is most undesirable to call this an extension, still less a completion, of quaternions.

The reader should be warned that the author often says "must" when there is no logical necessity at all. For instance, we are told that, β having one dimension in length, β^2 "must" have two; yet on the next page we are told that $\alpha\beta\gamma\delta$ means a solid angle, thus apparently having no dimensions in length, at any rate not four. This kind of fogginess is very common, even among quaternionists. Thus $ij=k$, so the product of two vectors can be a vector, and the law of dimensions is violated, or rather does not apply. Of course, in physics, it is convenient to represent areas, moments, &c., by vectors, and then the quaternion formulæ become more significant. We might, if we liked, put $ij=k_2$, $jk=i_2$, $ki=j_2$, regarding i_2, j_2, k_2 as areal units, and then have what Grassmann would call a regressive multiplication, $i_2j_2=k$, $j_2k_2=i$, $k_2i_2=j$, bringing us back to one dimension again. But anyone can see that this is unnecessary complication; in all physical applications of quaternions it is easy to see whether a vector is to be interpreted literally, or as the representative of some areal quantity.

Whatever may be the ultimate fate of this particular algebra, Dr. Macfarlane's researches deserve recognition. He has the spirit and the courage of a heretic, and every honest heretic helps to advance the truth.

G. B. M.

UNITED STATES METEOROLOGICAL PUBLICATIONS.[1]

(1) THE first thirty pages of the report of the Chief of the Weather Bureau for the year 1911–12 contain a summary of the work accomplished by that department during the year. This is followed by a general statement of the weather conditions prevailing in the individual months, while the last and by far the longest part of the report is devoted to tabulated statistics of the different meteorological elements with summaries of sunshine, excessive rainfall, &c.

An account of the work done at the upper-air station on Mount Weather is given first place in the volume, and from this we learn that it is proposed to modify the plan hitherto followed of attempting to obtain a kite or balloon flight on each day, regardless of the weather conditions, and to substitute a series of special ascents made to investigate particular problems. It is interesting to learn that a special department is being inaugurated at this observatory for the training of observers for duty at the 200 out-stations of the weather service. At the central office a synoptic weather chart is prepared each day for the whole of the northern hemisphere, and on this map are based general forecasts of the weather and temperature conditions over the United States for a week in advance. It is intended shortly to institute a service of wireless reports from ships in the Atlantic, and to transmit information as to the location and movements of dangerous storms to vessels from one of the high-power stations on the coast. Extensive observations are now being made on the snowfall of the western mountain ranges, and it is hoped to be able in the future to give useful forecasts of the flow of those rivers which are fed in the spring and summer by the thawing snow. A feature of the report is the list of new books added to the library during the year. Many of the more important of these works are referred to individually, and a short account is given of the scope covered by each book. This should prove useful for purposes of reference. It is evident from a perusal of the volume that the operations of the bureau are conducted on a very large scale, as befits an institution dealing with meteorological information from an area like that of the United States.

(2) The impending opening of the Panama Canal renders the subject of the second paper of especial importance at the present time. In addition to dealing with the West Indian hurricanes, the author sets out comparative data for the typhoons of the Pacific and the cyclones of the Bay of Bengal. All these disturbances are of the same type, characterised by a moderate decrease of atmospheric pressure to within forty or fifty miles of the centre, and then the rapid fall associated with the destructive winds which cause such havoc in the belt passed over by the central region of the disturbance. Nearly all the West Indian hurricanes have their origin in a well-marked area bounded by the parallels of 12° and 26° N. latitude, and lying between 56° and 90° W. longitude. The typical track is parabolic in shape, the storm moving W.N.W. at first, then curving round to the N., and finally passing in a north-easterly direction to the North Atlantic. The average rate of travel of these storms is only 300 miles per day, so that the forecaster is often enabled to give a fairly long warning of their approach. Much useful information is contained in the paper, and Dr. Fassig is to be congratulated on the completion of a trustworthy piece of work.

J. S. D.

[1] (1) Report of the Chief of the Weather Bureau, 1911–12.
(2) Hurricanes of the West Indies, Dr. O. L. Fassig.

REFLECTION AS A CONCEALING AND REVEALING FACTOR IN AQUATIC AND SUBAQUATIC LIFE.[1]

AS a result of observations and experiments carried out on ponds built for the purpose, and by the use of apparatus for observing organisms in their natural environments, I have arrived at certain conclusions as to the value of reflection as a concealing factor in various forms of aquatic and subaquatic life. The general principle upon which these ponds are built is as follows:—In one bank of the pond is a glass window, and beyond this window an underground observation chamber. No light enters this chamber except through the surface of the water. By this means everything in the pond is seen by *entirely* natural illumination, the observer cannot be detected, and as there is no reflection from the glass the making of photographic records is greatly simplified. In the first pond, built for the observation of objects in the water, the glass is perpendicular. In the second, for observing objects on the surface, the glass is at an angle of 45° to the surface.

Of apparatus I use a tube 18 in. square and 5 ft. long. On one side at the lower end is a window; into this tube slides a reflex camera, so that the lens is opposite the glass. When in use, a heavy weight carrying a hook is lowered into the water, with the end of the tube attached to the hook. The whole apparatus can be tilted at any angle, and by this means the incident rays from any object in any position—except overhead—are made to strike the glass at right angles, and thus distortion, due to refraction through the glass, is rendered negligible.

[1] Discourse delivered at the Royal Institution on Friday, June 6, by Dr. Francis Ward.